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Diffraction of a Gaussian laser beam by a straight edge leading to the formation of optical vortices and elliptical diffraction fringes

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ABSTRACT

The diffraction of a Gaussian laser beam by a straight edge has been studied theoretically and experimentally for many years. In this paper, we have experimentally observed for the first time the formation of the cusped caustic (for the Fresnel number $F \approx 100$) in the shadow region of the straight edge, with the cusp placed near the center of the circular laser beam($\lambda = 0.65$ μm) overlapped with the elliptical diffraction fringes. These fringes are originated at the region near the cusp of the caustic where light intensity is zero and the wave phase is singular (the optical vortex). We interpret observed diffraction fringes as a result of interference between the helical wave created by the optical vortex and cylindrical wave diffracted at the straight edge. We have theoretically revealed that the number of high contrast diffraction fringes observable in a shadow region is determined by the square of the diffracted angles in the range of spatial frequencies of the scattered light field in excellent agreement with experiments. The extra phase singularities with opposite charges are also observed along the shadow boundary as the fork-like diffraction fringes.

1. Introduction

This work was stimulated by the unusual experimental observation of the elliptical diffraction fringes in the shadow region of the straight edge by diffraction of the laser beam with a finite diameter.

The problem of light diffraction on half plane attracted the attention of theorists when using the diffraction theory proposed by Kirchhoff [1]. Sommerfeld first suggested the rigorous solution of such a problem [1,2]. Theoretical and experimental verification of the Kirchoff's approximation for a Gaussian beam with a finite diameter was done for the first time when studying diffraction on a half plane [3]. Another notable analysis was done in the work [4] using a converging beam. The authors of the works [5–7] theoretically investigated diffraction of a Gaussian beam on a half-plane. In the work [8] it was shown that at laser beam diffraction on a half-plane there arises an edge dislocation wave at the edge of the screen propagating in a shadow region. The structure of the field formed on diffraction of a Gaussian beam by an edge was studied rigorously in [9]. It was shown that the field diverging from the edge represents the superposition of cylindrical components, propagating as Young's boundary wave, produced by diffraction of the laser beam with a finite diameter.

However, such features as observation of the cusped caustic overlapped with high contrast elliptical diffraction fringes in the shadow of the straight edge, blocking more than a half of a circular laser beam, remained outside the scope of those papers.

In this paper, we have investigated the experimental conditions when a cusped caustic, an optical vortex and elliptical diffraction fringes occur in the shadow region of the straight edge shifted off the center of the circular laser beam. Within the edge-diffracted cusped caustic the necessary theoretical conditions [10–13] of the formation of the optical vortex are satisfied that leads to the formation of the diffracted wave front shaped as a helical surface. We have shown by numerical calculations that the Fresnel–Kirchhoff (FK) diffraction integral is not capable correctly represent the diffraction pattern consisting the edge-diffracted cusped caustic ($F \geq 100$) and the diffraction catastrophes' theory can be applied [13]. For Fresnel numbers $F \leq 30$ (the cusped caustic is not present), the numerical calculations by the FK integral produce diffraction patterns in good agreement with experimental one.

We may point out that the application of the FK integral rigorous solutions is not possible for the used experimental conditions.

1.1. Experiment

The experiment was performed using a circular laser beam with a small angular divergence, a beam diameter of 4 mm, and wavelength

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of 0.65 μm. One half or more of a circular laser beam was blocked by the screen edge. In the first series of experiments the diffraction pattern located at different distances from the screen edge was projected on the observation screen by a lens with magnification allowing to record an image of the diffraction fringes near the boundary of the lighted area. In the second series of experiments the diffraction was studied at distances of 15 and 60 cm from the screen edge. The screen edge was shifted off the center gradually to block an increasing part of the beam (see video in the supplemental material).

1.2. Numerical calculations

The Fresnel–Kirchhoff diffraction integral applied for calculation of the wave amplitude and phase at the point of observation is represented by the well-known formula [1]:

$$U\left(x_s, y_s, z_s\right) = -\frac{U_0 i}{2\lambda} \iint_{\mathrm{A}} \frac{\exp\left[ik\left(r+s\right)\right]}{r \cdot s} \left[\cos\left(\bar{n}, \bar{r}\right) - \left[\cos\left(\bar{n}, \bar{s}\right)\right]\right] dxdy \quad (1)$$

$U\left(x_s, y_s, z_s\right)$ — the complex amplitude at the point of observation; U_0 — the amplitude of the oscillations of the field; λ — wavelength; $k = 2\pi/\lambda$; $\bar{n}\left(0,0,1\right)$ is a unit vector normal to the plane of the aperture; xy — the coordinates of the aperture point.

The origin of the reference system is compatible with the aperture plane. Vectors $\bar{r}(0,0,1)$ and $\bar{s}(x_s, y_s, z)$, accordingly, determine the position of the light source and a point of observation.

Since the laser is used in experiments, a Gaussian amplitude distribution across the beam is applied:

$$U_G\left(x,y,z\right) = \frac{w_0}{w} \exp\left[-\frac{x^2+y^2}{w^2}\right] \times \exp\left[i(kz - \phi + \frac{\left(x^2+y^2\right)k}{2R})\right] \quad (2)$$

$$w = w_0\left[1 + \left(\frac{\lambda z_0}{\pi w_0^2}\right)^2\right]^{1/2}; R = z_0\left[1 + \left(\frac{\lambda z_0}{\pi w_0^2}\right)^{-2}\right]; \phi = arctg\left(\frac{\lambda z_0}{\pi w_0^2}\right)$$

w_0 — the minimum radius of the beam cross-section ; z_0 — the beam cross-section offset relative to the minimal cross-section.

Let the beam minimal cross-section coincides with the plane of the aperture $z_0 = 0$, and the amplitude of the beam at the point of observation $\left(x_s, y_s, z\right)$ can be represented as:

$$U\left(x_s, y_s, z_s\right) = -\frac{U_0 i}{2\lambda} \iint_{\mathrm{A}} \frac{1}{w_0} \exp\left[-\frac{x^2+y^2}{w_0^2}\right] \times \frac{\exp\left[iks\right]}{s}\left[-1 - \frac{\left(\bar{n}\cdot\bar{s}\right)}{|\bar{s}|}\right] dxdy. \quad (3)$$

Numerical integration (3) is performed by all cells of the partition of the aperture for each cell of the partition of the observation plane. The intensity and phase are calculated using the complex amplitude. The intensity is presented in relative units.

The integral has oscillating terms that lead to instability of the numerical solution. This problem was solved by increasing the number of nodes in the plane of the aperture. The number of nodes is selected depending on the size of the aperture and the distance to the observation plane. In particular, a satisfactory result (based on comparison with experiment) was obtained with a number of nodes greater than 400×400 for the aperture diameter $d = 4$ mm.

The intensity and phase are displayed using a gradient scale. The distribution of the intensity can vary over a wide range. In this case, the overexposure is applied for clarity. In particular, it is used to display the intensity in the area of the geometric shadow.

2. Results

In the first series of experiments, we observed the formation of the cusped caustic overlapped with the elliptical diffraction fringes in the shadow region of the edge. The cusped caustic was observed for the Fresnel number $F \approx 100$, 60 mm distance from the edge shifted off 1 mm from the beam center (an x axis direction), with the cusp placed near the center of the circular laser beam (shown in Fig. 1(a)). These results shows that the elliptical diffraction fringes start from the region near the cusp of the caustic where light intensity is zero and the wave phase is singular (the optical vortex).

The numerical calculations have shown that the FK diffraction integral is not capable correctly represent the diffraction pattern consisting the edge-diffracted cusped caustic and the diffraction catastrophes' theory should be applied in this case [13]. At the distances of 150 mm and longer from the aperture (the cusped caustic is not present), the numerical calculations by the FK integral produce diffraction patterns well correlated with experimental one. The example of the diffraction pattern recorded at the distance of 180 mm from the aperture is shown in Fig. 1(b) alongside with the numerical calculations shown in Fig. 1(c) and (d) (the 2D wave intensity and 2D phase). The FK theoretical pattern is in good agreement with the experimental one. These results shows that the elliptical diffraction fringes start from the region near the cusp of the caustic where light intensity is zero and the wave phase is singular (the optical vortex). The phase singularity (optical vortex) is clear observed in the 2D wave phase pattern (shown by the blue arrow) in the point where the wave intensity is zero. We interpret observed diffraction fringes as a result of interference between the helical wave created by the optical vortex and cylindrical wave diffracted at the edge of the screen.

In the diffraction catastrophes' theory the cusped caustic is classified as the Pearcey diffraction catastrophe [13]. The cusp is one of the caustics stable forms. We used the thin transparent sheet of the plastic placed in the laser beam to inset the strong phase perturbations in the laser wave front before diffraction by the straight edge. The resulting diffraction pattern is shown in Fig. 1(e). The elliptical diffraction fringes are practically vanished but the caustic's cusp has survived (shown by the blue arrow in Fig. 1(e)). Diffraction at the distance of 15 mm from the straight edge is shown in Fig. 1(f). A cylindrical edge dislocation (CED) wave [9] is visualized in the shadow transition region as dark and white diffraction fringes parallel to the straight edge (shown by the blue arrow). Few elliptical diffraction fringes are clearly observed in the central part of the shadow region due to interference of a helical wave and a CED wave.

The momentum flux carried by the laser beam is $\boldsymbol{g}_\perp(\boldsymbol{r}) \sim I(\boldsymbol{r})\nabla\varphi(\boldsymbol{r})$, where $I(\boldsymbol{r})$ is the light's intensity and $\varphi(\boldsymbol{r})$ is a transverse phase distribution of the laser field [14]. The beam of light with the helical phase distribution transports an orbital angular momentum (OAM) flux, $\boldsymbol{r} \times \boldsymbol{g}_\perp(\boldsymbol{r})$ [15].

We numerically calculated the momentum flux $\boldsymbol{g}_\perp(\boldsymbol{r})$ carried by the diffracted beam using the intensity and phase distribution data shown in Fig. 1(c) and (d). The calculated momentum flux pattern relating to the projection of the vector $\boldsymbol{g}_\perp(\boldsymbol{r})$ on x-axis is shown in Fig. 1(g) where the white–black color corresponds to the negative values of $\nabla_x\varphi(\boldsymbol{r})$ and the red color relates to the positive one. Yellow arrows shown in this figure visualize the $\boldsymbol{g}_\perp(\boldsymbol{r})$ in points of interest.

The circulation of the momentum flux along elliptical lines placed around the point with zero intensity is a direct evidence of the helical wave carrying the OAM. In addition, the cylindrical waves propagated in the shadow of the straight edge and in the illuminating beam area are clearly visualized.

Experiments and FK integral numerical calculations solved the problem of occurrence and number of high contrast diffraction fringes at the different distances ($z = 20, 30, 40, 50, 60$ cm) from the screen edge. The resulting diffraction patterns are presented in Fig. 2 alongside with the results of numerical calculations. The top horizontal row in Fig. 2 shows the experimental diffraction patterns. The second horizontal row from the top represents numerical calculations of the 2D intensity, and the third and fourth rows from the top show the 2D phase. The lines of the constant phase equal to (0 ± 0.1) radian are selected on the numeric 2D phase diagram (fourth row from the top in yellow, visualizing the 2D zero phase of the cylindrical wave). On the same chart, the spiral