Optics Communications 412 (2018) 80–84

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Fast, large-scale hologram calculation in wavelet domain

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ARTICLE INFO

Keywords:
Computer-generated hologram
Electroholography
Hologram
Holography
Holographic display

ABSTRACT

We propose a large-scale hologram calculation using WAvelet ShrinkAge-Based superpositIon (WASABI), a wavelet transform-based algorithm. An image-type hologram calculated using the WASABI method is printed on a glass substrate with the resolution of $65,536 \times 65,536$ pixels and a pixel pitch of 1 μm. The hologram calculation time amounts to approximately 354 s on a commercial CPU, which is approximately 30 times faster than conventional methods.

Holograms can be calculated by simulating a light wave emitted from three-dimensional (3D) objects. The development of an ideal 3D display can be expected when these holograms are displayed on a spatial light modulator (SLM) [1]. Unfortunately, this 3D display has the disadvantages that the size and viewing area of the 3D objects are small and narrow. This has been a hindrance to their practical use; however, in recent years, several techniques to address these issues have been proposed. A common factor in these techniques is that to observe the 3D objects with a wide viewing angle and large size by multiple observers, it is necessary to calculate a hologram with a large number of pixels and display it spatially [2–4] or temporally [5]. In other techniques, hologram printers capable of printing large-scale holograms with minute pixel pitch have been actively employed to reproduce 3D objects with a wide viewing area and large size [6–10].

The hologram calculation time for reconstructing 3D objects with wide viewing angles and large size is considerably lengthy. Various fast hologram calculation methods have been proposed with procedures of 3D object expression. In the case of 3D objects represented by polygons, tilted diffraction calculation [11] can effectively compute a hologram. A holographic stereogram [12–15] can be used when 3D objects are expressed in multi-view images. In the case that 3D objects are represented by an RGB-D image, a hologram calculation using diffraction calculation of cross sectional images generated from the RGB-D images has been proposed [16].

For 3D objects represented by point light sources, a hologram can be calculated by adding light waves from each point light source on the hologram plane. Many acceleration methods for the point light source model have been proposed, such as an image hologram method [17], look-up table methods (LUT) [18,19], and wavefront recording plane methods [20,21]. Recently, we proposed WAvelet ShrinkAge-Based superpositIon (WASABI), a wavelet transform-based method [22,23]. This method superimposes light waves of point light sources in a wavelet domain. However, although the WASABI method has shown that hologram calculation can be efficiently performed using a small-scale hologram of $2,048 \times 2,048$ pixels, thus far, the implementation of large-scale hologram calculations has not been accomplished.

In this study, we generate a large-scale hologram calculation using the WASABI method. An image-type hologram calculated using the WASABI method is printed on a glass substrate. The resolution of the hologram is $65,536 \times 65,536$ pixels with the pixel pitch of 1 μm. The calculation time of the hologram is approximately 354 s on a commercial CPU. The calculation is capable of a 30-factor acceleration compared to conventional methods.

1. Large-scale hologram calculation using WASABI method

A hologram can be calculated by adding light waves emitted from each object point on the hologram plane. The following equation is used for the calculation:

$$u(x_h, y_h) = \sum_j^N a_j \exp(i\frac{2\pi}{\lambda} r_{hj}) = \sum_j^N a_j u_{z_j}(x_h - x_j, y_h - y_j), \tag{1}$$

where $i = \sqrt{-1}$, N is the total number of the point light sources, (x_j, y_j, z_j) and a_j are the coordinate and the amplitude of jth point

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https://doi.org/10.1016/j.optcom.2017.11.066
Received 13 August 2017; Received in revised form 23 November 2017; Accepted 23 November 2017

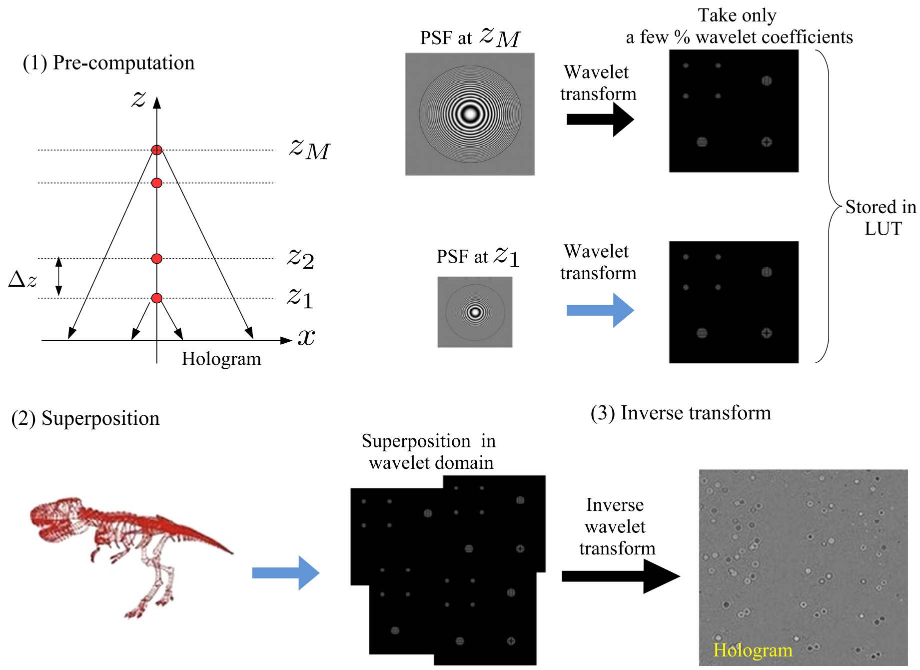

Fig. 1. Calculation procedure of the WASABI method.

light source, respectively. (x_h, y_h) and (x_j, y_j) are normalized by the pixel pitch of the hologram, p. r_{hj} is calculated by $r_{hj} = \sqrt{p^2(x_h - x_j)^2 + p^2(y_h - y_j)^2 + z_j^2}$. z_j is quantized by a constant Δz. λ is the wave length, r_{hj} is the distance between jth object point and (x_h, y_h), and u_{z_j} is the point spread function (PSF) of a point located in $(0, 0, z_j)$. Although this calculation is simple, when the PSF is distributed over the entire hologram, the calculation complexity is $O(NN_h^2)$ where the number of pixels of the hologram is $N_h \times N_h$. Therefore, this accounts for the greatest calculating cost among hologram calculations.

The WASABI method can reduce this computation amount via applying the wavelet transform to the PSF and by superposing the PSFs in the wavelet domain [22]. Fig. 1 shows the calculation steps of the WASABI method. The calculation steps are explained as follows:

(1) Pre-computation: We pre-compute PSFs by $u_{z_j} = \exp(ikr_{hj})\,\mathrm{circ}(\sqrt{x_h^2 + y_h^2}/(2W_j))$ where W_j is the PSF radius of the jth object point and $\mathrm{circ}(c) = 1$ when $c < 1$, otherwise 0. The depth coordinate of the pre-calculated PSFs is quantized by $\Delta z = z_{j+1} - z_j$ and the PSFs are converted to wavelet-domain PSFs using fast wavelet transform (FWT). We select N_γ strong coefficients among the wavelet-domain PSF and store them in the LUT. In this study, N_γ is determined by $N_\gamma = \pi(W_j/2)^2\gamma$ where γ is the selectivity. If Δz is not a constant, it is necessary to store PSFs of any distances, which require a huge amount of memory. Therefore, by setting Δz as a constant, we can reduce the amount of memory [19].
(2) Superposition: The superposition corresponding to Eq. (1) in the wavelet domain is performed using the coefficients stored in the LUT.
(3) Inverse transformation: The superposed result is converted to the space domain using the inverse FWT.

As shown in Fig. 2(a), a wavelet-domain PSF is expressed by scaling coefficients $s_{m,n}^{(\ell)}$ and three wavelet coefficients $w_{LH,m,n}^{(\ell)}$, $w_{HL,m,n}^{(\ell)}$ and $w_{HH,m,n}^{(\ell)}$, where ℓ denotes the level of FWT and the subscripts L and H denote low and high frequencies, respectively. Fig. 2(a) shows an example of a wavelet-domain PSF at level 2. In this study, we used coiflets, which are one of the discrete wavelet transforms, as the wavelets at level 3. Coiflets have a more symmetrical shape compared to Daubechies wavelets. We used a PSF with a wider spatial bandwidth product than Ref. [22], so that the Daubechies wavelet of level 2 could not express such a PSF well. We adopted FWT and IFWT with coiflets of level 3 in consideration of the calculation time of a hologram and image quality of a reconstructed image.

The superposition in the wavelet domain is performed by

$$\psi(m, n) = \sum_{j=0}^{N} a_j \sum_{k=0}^{N_\gamma - 1} c_{z_j,k}\,\delta(m - \alpha_{z_j,k} x_j, n - \alpha_{z_j,k} y_j), \tag{2}$$

where $\psi(m, n)$ is the superposition result of PSFs in the wavelet domain, (m, n) is the coordinate in the wavelet domain, $\delta(m, n)$ is the Dirac delta function and $c_{z_j,k} \in \{s_{m,n}^{(\ell)}, w_{LH,m,n}^{(\ell)}, w_{HL,m,n}^{(\ell)}, w_{HH,m,n}^{(\ell)}\}$ is the N_γ strong coefficients and $\alpha_{z_j,k}$ is the shift weight according to the level ℓ of the FWT. Eq. (2) means that the amplitude of an object point is multiplied by the wavelet coefficient of the PSF corresponding the object point, and the result is superimposed on $\psi(m, n)$.

When calculating a large-scale hologram by the WASABI method, we first divide an entire hologram into sub-holograms, as shown in Fig. 2(b). This division can considerably reduce the memory usage of a computer as compared with calculating the entire hologram at once. Herein, the number of pixels of the entire hologram is $N_w \times N_w$, and the number of pixels for each sub hologram is $N_h \times N_h$. The index of the sub holograms currently being calculated is represented by (s, t). The calculation performed is explained as follows:

(1) Pre-computation: As described above, PSFs for all depths are converted to the wavelet domain via FWT. These PSFs $u_{z_j}(x_h, y_h)$ are calculated within the area $|x_h| < N_h/2$ and $|y_h| < N_h/2$. N_γ strong coefficients are stored in the LUT.
(2) Coordinate transformation of object points: The coefficients after FWT are defined for sub-holograms ($N_h \times N_h$ pixels), and not for the entire hologram ($N_w \times N_w$ pixels). Therefore, the positions of all the object points are shifted relative to the current sub-hologram (s, t). Thus, the shift is given as $x'_j = x_j - sN_h$ and $y'_j = y_j - tN_h$.
(3) Superposition: Using x'_j and y'_j instead of x_j and y_j in Eq. (2), the superposition is performed by $\psi(m, n) = \sum_{j=0}^{N} a_j \sum_{k=0}^{N_\gamma-1} c_{z_j,k}\delta(m - \alpha_{z_j,k} x'_j, n - \alpha_{z_j,k} y'_j)$.