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Freezing optical rogue waves by Zeno dynamics

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A B S T R A C T

We investigate the Zeno dynamics of the optical rogue waves. Considering their usage in modeling rogue wave dynamics, we analyze the Zeno dynamics of the Akhmediev breathers, Peregrine and Akhmediev–Peregrine soliton solutions of the nonlinear Schrödinger equation. We show that frequent measurements of the wave inhibits its movement in the observation domain for each of these solutions. We analyze the spectra of the rogue waves under Zeno dynamics. We also analyze the effect of observation frequency on the rogue wave profile and on the probability of lingering of the wave in the observation domain. Our results can find potential applications in optics including nonlinear phenomena.

1. Introduction

Rogue (freak) waves can be described as high amplitude waves with a height bigger than 2–2.2 times the significant waveheight in a wavefield. Their studies have become extensive in recent years [1–4]. The research has emerged with the investigation of one of the simplest nonlinear models, which is the nonlinear Schrödinger equation (NLSE) [1]. Discovery of the unexpected rogue wave solutions of the NLSE resulted in seminal studies of rogue wave dynamics, such as in Ref. [1]. Their existence is not necessarily restricted to optical media [5], they can also be observed in hydrodynamics, Bose–Einstein condensation, acoustics and finance, just to name a few [1,2]. It is natural to expect that in a medium whose dynamics are described by the NLSE and NLSE like equations, rogue waves can also emerge. In this study we consider optical rogue waves for which analyzing the dynamics, shapes and statistics of rogue wavy optical fields are crucially important to satisfy certain power and communication constraints.

On the other hand, quantum Zeno dynamics [6,7], which is the inhibition of the evolution of an unstable quantum state by appropriate frequent observations during a time interval has attracted an intense attention in quantum science, usually for protecting the quantum system from decaying due to inevitable interactions with its environment. It emerged that the observation alters the evolution of an atomic particle, even can stop it [8–10]. Duan and Guo showed that the dissipation of two particles can be prevented [11,12], Viola and Lloyd proposed a dynamical suppression of decoherence of qubit systems [13], Maniscalco et al. proposed a strategy to fight against the decoherence of the entanglement of two atoms in a lossy resonator [14], Nourmandipour et al. studied Zeno and anti-Zeno effects on the entanglement dynamics of dissipative qubits coupled to a common bath [15], and Bernu et al. froze the coherent field growth in a cavity [16]. Very recently, Facchi et al. studied the large-time limit of the quantum Zeno effect [17].

Quantum Zeno dynamics can also be used for realizing controlled operations and creating entanglement. Creation of entanglement is a major issue in quantum information science, requiring controlled operations such as CNOT gates between qubits, which is usually a demanding task. As the number of qubits exceeds two, multipartite entanglement emerges in inequivalent classes such as GHZ, W and cluster states, which cannot be transformed into each other via local operations and classical communications. The preparation of multipartite entangled states – especially W states – require not only even more controlled operations but also novel methods [18–22]. Wang et al. proposed a collective threshold measurement scheme for creating bipartite entanglement, avoiding the difficulty of applying CNOT gates or performing Bell measurements [23], which can be extended to multipartite entangled states. Chen et al. proposed to use Zeno dynamics for generation of W states robust against decoherence and photon loss [24] and Barontini et al. experimentally demonstrated the deterministic generation of W states by quantum Zeno dynamics [25]. Nakazato et al. further showed that purifying quantum systems is possible via Zeno-like measurements [26].

Optical analogue of the quantum Zeno effect has been receiving an increasing attention. Yamane et al. reported Zeno effect in optical fibers [27]. Longhi proposed an optical lattice model including

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tunneling-coupled waveguides for the observation of the optical Zeno effect [28]. Leung and Ralph proposed a distillation method for improving the fidelity of optical Zeno gates [29] Biagioni et al. experimentally demonstrated the optical Zeno effect by scanning tunneling optical microscopy [30]. Abdullaev et al. showed that it is possible to observe the optical analog of not only linear but also nonlinear quantum Zeno effects in a simple coupler and they further proposed a setup for the experimental demonstration of these effects [31]. McCusker et al. utilized quantum Zeno effect for the experimental demonstration of the interaction-free all-optical switching [32]. Thapliyal et al. studied quantum Zeno and anti-Zeno effects in nonlinear optical couplers [33].

In this paper we numerically investigate the optical analogue of quantum Zeno dynamics of the rogue waves that are encountered in optics. With this motivation, in the second section of this paper we review the NLSE and the split-step Fourier method for its numerical solution. We also review a procedure applied to wavefunction to model the Zeno dynamics of an observed system. In the third section of this paper, we analyze the Zeno dynamics of the Akhmediev breathers, Peregrine and Akhmediev–Peregrine soliton solutions of the NLSE, which are used as models to describe the rogue waves. We show that frequent measurements of the wave inhibits the movement of the wave in the observation domain for each of these types of rogue waves. We also analyze the spectra of the rogue waves under Zeno dynamics and discuss the effect of observation frequency on the rogue wave profile and on the probability of lingering of the wave in the observation domain. In the last section we conclude our work and summarize future research tasks.

2. Nonlinear Schrödinger equation and Zeno effect

It was shown that all the features of linear quantum mechanics can be reproduced by NLSE [34], and quantum NLSE can accurately describe quantum optical solitons in photonic waveguides with Kerr nonlinearity [35–38]. The bosonic matter wave field for weakly interacting ultracold atoms in a Bose–Einstein condensate, evolves according to quantum NLSE [39,40]. Many nonlinear phenomena observed in fiber optics are generally studied in the frame of the NLSE [1]. Optical rogue waves are one of those phenomena and rational rogue wave soliton solutions of the NLSE are accepted as accurate optical rogue wave models [1]. In order the analyze the Zeno dynamics of rogue waves, we consider the nondimensional NLSE given as

$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2\psi = 0, \tag{1}$$

where x and t are the spatial and temporal variables, respectively, i is the imaginary number, and ψ is the complex amplitude. It is known that the NLSE given by Eq. (1) admits many different types of analytical solutions. Some of these solutions are reviewed in the next section of this paper. For arbitrary wave profiles, where the analytical solution is unknown, the NLSE can be numerically solved by a split-step Fourier method (SSFM), which is one of the most commonly used forms of the spectral methods. Similar to other spectral methods, the spatial derivatives are calculated using spectral techniques in SSFM. Some applications of the spectral techniques can be seen in Refs. [41–56] and their more comprehensive analysis can be seen in Refs. [57–60].

The temporal derivatives in the governing equations is calculated using time integration schemes such as Adams–Bashforth and Runge–Kutta, etc. [47,59,60]. However, SSFM uses an exponential time stepping function for this purpose. SSFM is based on the idea of splitting the equation into two parts, namely the linear and the nonlinear parts. Then time stepping is performed starting from the initial conditions. In a possible splitting we take the first part of the NLSE as

$$i\psi_t = -|\psi|^2\psi \tag{2}$$

which can exactly be solved as

$$\tilde{\psi}(x, t_0 + \Delta t) = e^{i|\psi(x,t_0)|^2 \Delta t}\, \psi_0, \tag{3}$$

where Δt is the time step and $\psi_0 = \psi(x, t_0)$ is the initial condition. The second part of the NLSE can be written as

$$i\psi_t = -\frac{1}{2}\psi_{xx}. \tag{4}$$

Using a Fourier series expansion we obtain

$$\psi(x, t_0 + \Delta t) = F^{-1}\left[e^{-ik^2/2\Delta t} F[\tilde{\psi}(x, t_0 + \Delta t)]\right], \tag{5}$$

where k is the wavenumber [44,45]. Substituting Eq. (3) into Eq. (5), the final form of the SSFM becomes

$$\psi(x, t_0 + \Delta t) = F^{-1}\left[e^{-ik^2/2\Delta t} F[e^{i|\psi(x,t_0)|^2 \Delta t}\, \psi_0]\right]. \tag{6}$$

Starting from the initial conditions, the time integration of the NLSE can be done by the SSFM. Two fast Fourier transform (FFT) operations per time step are needed for this form of the SSFM. The time step is selected as $\Delta t = 10^{-3}$, which does not cause a stability problem. The number of spectral components are taken as $M = 2048$ in order to use the FFT routines efficiently.

Although it is known that the decay of an atomic particle can be inhibited by Zeno dynamics, it remains an open question whether the rogue waves in the quantized optical fields in the frame of the NLSE can be stopped by Zeno dynamics. In this paper we analyze the Zeno dynamics of such rogue waves by using the SSFM reviewed above. Although analytical solution of the NLSE is known and used as initial conditions in time stepping of SSFM, after a positive Zeno measurement the wavefunction becomes complicated thus numerical solution is needed. Recently a theoretical wavefunction formulation of the quantum Zeno dynamics is proposed in Ref. [61], used in Refs. [62,63] and experimentally tested in Ref. [64]. In this formulation, after a positive measurement the particle is found in the observation domain of $[-L, L]$ with a wavefunction of $\psi_T(x,t) = \psi(x,t)\mathrm{rect}(x/L)/\sqrt{P}$ where $P = \int_{-L}^{L} |\psi(x,t)|^2 dx$, and $\mathrm{rect}(x/L) = 1$ for $-L \le x \le L$, and 0 elsewhere [61]. Between two successive positive measurements, the wave evolves according to NLSE. This cycle can be summarized as

$$\psi_T\left(x, \frac{(n-1)t}{N}\right) \overset{evolve}{\rightarrow} \psi\left(x, \frac{nt}{N}\right) \overset{measure}{\rightarrow}$$
$$\psi_T\left(x, \frac{nt}{N}\right) = \psi\left(x, \frac{nt}{N}\right)\frac{\mathrm{rect(x/L)}}{\sqrt{P_N^n}} \tag{7}$$

where n is the observation index, N is the number of observations [61], and

$$P_N^n = \int_{-L}^{L} \left|\psi\left(x, \frac{nt}{N}\right)\right|^2 dx. \tag{8}$$

The cumulative probability of finding the wave in the observation domain becomes [61]

$$P_N = \prod_{n=1}^{N} P_N^n. \tag{9}$$

Using the momentum representation of the linear Schrödinger equation and analogy of optical wave dynamics of Fabry–Perot resonator, an analytical derivation of the lingering probability of an atomic particle in the interval of $[-L, L]$ after nth measurement is given as

$$P_N^n \approx 1 - 0.12\left(\frac{4}{\pi}\right)^2\left(\frac{2\pi t}{N}\right)^{3/2} \tag{10}$$

in [61]. After N measurements the cumulative probability of finding the particle in the observation domain becomes

$$P_N \approx \left(1 - 0.12\left(\frac{4}{\pi}\right)^2\left(\frac{2\pi t}{N}\right)^{3/2}\right)^N \tag{11}$$

which can further be simplified using Newton's binomial theorem [61]. The reader is referred to Ref. [61] for the details of the derivation of these relations. We compare the analytical relations given in Eqs. (10)–(11) with the numerical probability calculations in the next section of this paper. By applying this procedure we show that the evolution of