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Steady bound electromagnetic eigenstate arises in a homogeneous isotropic linear metamaterial with zero-real-part-of-impedance and nonzero-imaginary-part-of-wave-vector

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ABSTRACT

In this paper, we shall demonstrate theoretically that steady bound electromagnetic eigenstate can arise in an infinite homogeneous isotropic linear metamaterial with zero-real-part-of-impedance and nonzero-imaginary-part-of-wave-vector, which is partly attributed to that, here, nonzero-imaginary-part-of-wave-vector is not involved with energy losses or gain. Altering value of real-part-of-impedance of the metamaterial, the bound electromagnetic eigenstate may become to be a progressive wave. Our work may be useful to further understand energy conversion and conservation properties of electromagnetic wave in the dispersive and absorptive medium and provides a feasible route to stop, store and release electromagnetic wave (light) conveniently by using metamaterial with near-zero-real-part-of-impedance.

1. Introduction

It is well known that, in a source-free infinite homogeneous linear medium, time-harmonic electromagnetic fields will usually behavior as a progressive wave [1]. Localized state of electromagnetic wave can be achieved by applying disordered system [2], micro- or macro-optic resonator [3], waveguide structure [4–7] and so on, which may be used to slow or even stop light, and is believed to be an attractive technique for enhanced nonlinear optics [5], light harvesting [6], and optical (quantum) signal processing [7]. Nowadays the study of localized states of electromagnetic wave has been attracted more attention [2–9].

Theoretically prediction [10] and experimental verification [11] of negative refraction of electromagnetic waves at an interface formed by left-handed material (LHM) and usual right-handed material (RHM) arouse great interest in designing and realizing metamaterials with unconventional values of electromagnetic parameters, discovering novel electromagnetic phenomena and developing potential applications [8,9,12–15]. One of the most interesting applications of metamaterials is used to stop and even store light by designing appropriate hetero-structures [4–8]. Recently, it is predicted theoretically that electromagnetic wave may stop steadily in an active medium with zero-real-part-of-impedance, which corresponds to the case that the time-dependent Poynting vector (TDPV) shift forward and then turn backward periodically [9]. However, there is a lack of detailed study on characteristics of electromagnetic wave in the medium with near-zero-real-part-of-impedance, and hence, there is no feasible way to be provided to carry out the manipulation of electromagnetic wave capture, storage and release.

In this work, based on eigenstate of source-free electromagnetic wave equations, we shall address energy conversion and conservation properties of electromagnetic wave in the dispersive and absorptive medium, and demonstrate that steady bound electromagnetic eigenstate can arise in a metamaterial with zero-real-part-of-impedance and nonzero-imaginary-part-of-wave vector. Altering value of real-part-of-impedance of the metamaterial, the bound electromagnetic eigenstate may become to be either a forward or a backward progressive wave depending on initial conditions and sign of real-part-of -impedance. These results may provide a feasible route to stop electromagnetic wave completely, store electromagnetic wave for a long time, and control propagation direction of electromagnetic wave conveniently. The remainder of the paper is organized as follows: In Section 2, theoretical analyses are presented. In Section 3, the realizable experiments are suggested to test the theories. Finally, some conclusions are drawn in Section 4.

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2. Theoretical analyses

Let us begin by briefly reviewing some fundamental knowledge of electromagnetic fields and waves, which is useful for us to clearly demonstrate properties of electromagnetic eigenstate in a homogeneous isotropic linear medium with near-zero-real-part-of-impedance. Choosing time dependence of $e^{i\omega t}$, the homogeneous isotropic linear medium can be represented by a complex scalar relative permittivity $\widetilde{\varepsilon} = |\widetilde{\varepsilon}_r|\varepsilon_0 \exp(-i\alpha_\varepsilon) = \varepsilon' - i\varepsilon''$ and permeability $\widetilde{\mu} = |\widetilde{\mu}_r|\mu_0 \exp(-i\alpha_\mu) = \mu' - i\mu''$, respectively. Here, the complex valued parameters are marked with superscript "~". $\alpha_{\varepsilon(\mu)}$ is electric (magnetic) damping angle. For passive media, both α_ε and α_μ are limited in the range of $[0, \pi]$, and for active media, at least one of the two damping angles of α_ε and α_μ will certainly fall in the range of $(\pi, 2\pi)$ [9,16]. The source-free electromagnetic wave equation is written as

$$\nabla^2 \vec{\widetilde{E}} - \widetilde{\varepsilon}\widetilde{\mu}\frac{\partial^2 \vec{\widetilde{E}}}{\partial t^2} = 0. \tag{1}$$

Where, $\vec{\widetilde{E}}$ is complex valued electric field intensity vector. It is a three-dimensional form of the wave equation. Considering a uniform plane wave characterized by a uniform $\vec{\widetilde{E}}_x$ (uniform magnitude and constant phase) over the plane surfaces perpendicular to z, that is $\partial^2 \vec{\widetilde{E}}/\partial x^2 = 0$ and $\partial^2 \vec{\widetilde{E}}/\partial y^2 = 0$. Eq. (1) simplifies to

$$\frac{\partial^2 \widetilde{E}_x}{\partial z^2} - \widetilde{\varepsilon}\widetilde{\mu}\frac{\partial^2 \widetilde{E}_x}{\partial t^2} = 0. \tag{2}$$

The eigenstate of electromagnetic wave equation (2) takes the form:

$$\begin{aligned}\widetilde{E}_x(z,t) &= \widetilde{E}_{x,+}(z,t) + \widetilde{E}_{x,-}(z,t)\\ &= \widetilde{E}_{x,+}(z_0,0)e^{i\omega t - i\widetilde{k}(z-z_0)} + \widetilde{E}_{x,-}(z_0,0)e^{i\omega t + i\widetilde{k}(z-z_0)}.\end{aligned} \tag{3}$$

Where, $\widetilde{E}_{x,+}(z,t)(\widetilde{E}_{x,-}(z,t))$ usually refers to electromagnetic wave traveling along the $+z$ $(-z)$ direction, $\widetilde{E}_{x,+}(z_0,0)$ and $\widetilde{E}_{x,-}(z_0,0)$ are arbitrary (and, in general, complex) constants that must be determined by initial and/or boundary conditions, $\widetilde{k} = k' - ik'' = \omega\sqrt{|\widetilde{\varepsilon}\widetilde{\mu}|}e^{-i\alpha_k}$ ($\alpha_k = \frac{\alpha_\varepsilon + \alpha_\mu}{2}$) is the complex valued wave number. Correspondingly, the magnetic field can be given as

$$\widetilde{H}_y(z,t) = \widetilde{H}_{y,+}(z,t) + \widetilde{H}_{y,-}(z,t) = \frac{\widetilde{E}_{x,+}(z,t)}{\widetilde{\eta}} - \frac{\widetilde{E}_{x,-}(z,t)}{\widetilde{\eta}}. \tag{4}$$

Where, $\widetilde{\eta} = \eta' - i\eta'' = \sqrt{|\widetilde{\mu}/\widetilde{\varepsilon}|}e^{-i\alpha_\eta}$ ($\alpha_\eta = \frac{\alpha_\mu - \alpha_\varepsilon}{2}$) is impedance of the medium.

To address energy conversion and conservation properties of electromagnetic wave traveling in a medium, the Poynting theorem is adopted [1,9]

$$\vec{J} \cdot \vec{E} = -\nabla \cdot (\vec{E} \times \vec{H}) - (\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}). \tag{5}$$

Where, $\vec{J}$ is electric current density, $\vec{E} \equiv \mathrm{Re}(\vec{\widetilde{E}})$, $\vec{B} \equiv \mathrm{Re}(\vec{\widetilde{B}})$, $\vec{H} \equiv \mathrm{Re}(\vec{\widetilde{H}})$ and $\vec{D} \equiv \mathrm{Re}(\vec{\widetilde{D}})$ are real valued electric field intensity, magnetic flux density, magnetic field intensity and electric flux density, respectively. Assuming $\vec{J} \cdot \vec{E} = 0$, i.e., energy losses induced by electric current is neglected. $\vec{S} \equiv \vec{E} \times \vec{H}$ is TDPV, $W_{e,m} \equiv -(\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t})$ can be taken as part of time-rate of work done by Lorentz force [9] and relates to storied and lossy energy densities [1,9]. According to Eqs. (3) and (4), $\vec{S}(z,t)$ and $W_{e,m}(z,t)$ are, respectively, obtained as

$$\begin{aligned}\vec{S}(z,t) &= \frac{E_{x,+}^2(z_0,0)e^{-2k''(z-z_0)}}{2|\widetilde{\eta}|}[\cos(2\omega t - 2k'(z-z_0) + \alpha_\eta) + \cos\alpha_\eta]\hat{e}_z\\ &+ \frac{E_{x,-}^2(z_0,0)e^{2k''(z-z_0)}}{2|\widetilde{\eta}|}[\cos(2\omega t + 2k'(z-z_0) + \alpha_\eta) + \cos\alpha_\eta](-\hat{e}_z)\\ &\equiv \vec{S}_+(z,t) + \vec{S}_-(z,t),\end{aligned} \tag{6}$$

$$\begin{aligned}W_{e,m}(z,t) &= \omega|\widetilde{\varepsilon}|E_{x,+}^2(z_0,0)e^{-2k''(z-z_0)}[\sin(2\omega t - 2k'(z-z_0) - \alpha_\varepsilon)\\ &- 0.5(\sin\alpha_\varepsilon + \sin\alpha_\mu)]\\ &+ \omega|\widetilde{\varepsilon}|E_{x,-}^2(z_0,0)e^{2k''(z-z_0)}[\sin(2\omega t + 2k'(z-z_0) - \alpha_\varepsilon)\\ &- 0.5(\sin\alpha_\varepsilon + \sin\alpha_\mu)]\\ &\equiv W_+(z,t) + W_-(z,t).\end{aligned} \tag{7}$$

The time-periodic terms in Eq. (7) indicate that energies are stored and then released by turns, thus these terms relate to stored energies. The time-independent terms in Eq. (7) correspond to energy losses ($\sin\alpha_\varepsilon + \sin\alpha_\mu > 0$) or gain ($\sin\alpha_\varepsilon + \sin\alpha_\mu < 0$) [9]. On the other hand, the divergence of TDPV can be derived from Eq. (6) as

$$\begin{aligned}\nabla \cdot \vec{S}(z,t) &= \nabla \cdot \vec{S}_+(z,t) + \nabla \cdot \vec{S}_-(z,t)\\ &= \frac{E_{x,+}^2(z_0,t)e^{-2k''(z-z_0)}}{2|\widetilde{\eta}|}[-2k''\cos(2\omega t - 2k'(z-z_0) + \alpha_\eta)\\ &+2k'\sin(2\omega t - 2k'(z-z_0) + \alpha_\eta) - 2k''\cos\alpha_\eta]\\ &+\frac{E_{x,-}^2(z_0,t)e^{2k''(z-z_0)}}{2|\widetilde{\eta}|}[-2k''\cos(2\omega t + 2k'(z-z_0) + \alpha_\eta)\\ &+2k'\sin(2\omega t + 2k'(z-z_0) + \alpha_\eta) - 2k''\cos\alpha_\eta].\end{aligned} \tag{8}$$

Noting relations of $\cos\alpha_k \sin(2\omega t - 2k'(z-z_0) + \alpha_\eta) - \sin\alpha_k \cos(2\omega t - 2k'(z-z_0) + \alpha_\eta) = \sin(2\omega t - 2k'(z-z_0) - \alpha_\varepsilon)$ and $2\cos\alpha_k \cos\alpha_\eta = \sin\alpha_\varepsilon + \sin\alpha_\mu$, it is found from Eqs. (7) and (8) that there are $\nabla \cdot \vec{S}(z,t) = W_{e,m}(z,t)$. Apparently, the term of $k''\cos\alpha_\eta$ in Eq. (8) corresponds to the term of $\sin\alpha_\varepsilon + \sin\alpha_\mu$ in Eq. (7), thus relates to energy losses (or gain). We shall point out that, for the special case of $\cos\alpha_\eta = 0$, there are $\sin\alpha_\varepsilon + \sin\alpha_\mu = 2\cos\alpha_k \cos\alpha_\eta = 0$ certainly, clearly, here, nonzero k'' is not involved with energy losses or gain, i.e., *nonzero k'' is not certain to relate to energy losses or gain*. In addition, terms of $k''\cos(2\omega t - 2k'(z-z_0) + \alpha_\eta)$ and $k''\cos(2\omega t + 2k'(z-z_0) + \alpha_\eta)$, as like as $k'\sin(2\omega t - 2k'(z-z_0) + \alpha_\eta)$ and $k'\sin(2\omega t + 2k'(z-z_0) + \alpha_\eta)$, relate to the stored energies, which indicates that nonzero k'' can affect distribution of stored energies, and thus influence distribution of amplitudes of electric and magnetic fields.

Since direction of TDPV usually oscillate with time except for the special cases of $\cos\alpha_\eta = \pm 1$, we consider the time-averaged Poynting vector (TAPV), which can be obtained from Eq. (6) as

$$\begin{aligned}\langle\vec{S}(z)\rangle &= \frac{E_{x,+}^2(z_0,0)e^{-2k''(z-z_0)}}{2|\widetilde{\eta}|}\cos\alpha_\eta \hat{e}_z\\ &+ \frac{E_{x,-}^2(z_0,0)e^{2k''(z-z_0)}}{2|\widetilde{\eta}|}\cos\alpha_\eta(-\hat{e}_z)\\ &\equiv \langle\vec{S}_+(z)\rangle + \langle\vec{S}_-(z)\rangle\end{aligned} \tag{9}$$

According to Eq. (6), phase difference α_{E-H} between $\widetilde{E}_{x,+}(z,t)$ and $\widetilde{H}_{y,+}(z,t)$ is $\alpha_{E-H} = \alpha_\eta$, and phase difference α_{E-H} between $\widetilde{E}_{x,-}(z,t)$ and $\widetilde{H}_{y,-}(z,t)$ can be taken as $\alpha_{E-H} = \pm\pi + \alpha_\eta$. Combining Eq. (9), we can say that TAPV direction depends on the sign of $\cos\alpha_{E-H}$, thus the value of phase difference α_{E-H} can be used to determine TAPV direction conveniently. Furthermore, inner product of TAPV and wave vector can be derived as [17]

$$\begin{aligned}\langle\vec{S}(z)\rangle \cdot \vec{k'} &= \frac{1}{2}\mathrm{Re}(\sqrt{\widetilde{\mu}/\widetilde{\varepsilon}})\mathrm{Re}(\omega\sqrt{\widetilde{\mu}\,\widetilde{\varepsilon}})[e^{-2k''(z-z_0)}|\frac{\widetilde{E}_{x,+}(z_0,0)}{\widetilde{\eta}}|^2\\ &+e^{2k''(z-z_0)}|\frac{\widetilde{E}_{x,-}(z_0,0)}{\widetilde{\eta}}|^2]\\ &\propto \cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2})\cos(\frac{\alpha_\mu + \alpha_\varepsilon}{2}).\end{aligned} \tag{10}$$

Apparently, direction relation between TAPV and wave vector depends on the sign of $\cos\alpha_\eta \cos\alpha_k$. Furthermore, combining Eqs. (3) and (4), change of amplitudes of electric and magnetic field intensities along TAPV direction can be determined subsequently.

Effects of dispersion can be accounted by adopting the signal spectrum consisted of two discrete different frequencies [9,18], it has been verified that the cross terms of both the Poynting vector and $W_{e,m}(z,t)$ associated with different frequencies average to be zero [9,18]. Therefore,