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Nonclassical thermal-state superpositions: Analytical evolution law and decoherence behavior

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ABSTRACT

Employing the integration technique within normal products of bosonic operators, we present normal product representations of thermal-state superpositions and investigate their nonclassical features, such as quadrature squeezing, sub-Poissonian distribution, and partial negativity of the Wigner function. We also analytically and numerically investigate their evolution law and decoherence characteristics in an amplitude-decay model via the variations of the probability distributions and the negative volumes of Wigner functions in phase space. The results indicate that the evolution formulas of two thermal component states for amplitude decay can be viewed as the same integral form as a displaced thermal state $\rho(V,d)$, but governed by the combined action of photon loss and thermal noise. In addition, the larger values of the displacement d and noise V lead to faster decoherence for thermal-state superpositions.

1. Introduction

In quantum optics, coherent- and thermal-state superpositions are regarded as the superpositions of the most classical states. They usually always exhibit some strong nonclassical properties and have many practical applications in quantum information processing, such as in quantum superpositions and other relevant problems (e.g., nonclassicality and decoherence, etc.) that are currently topics of intense interest by physicists.

A superposition of coherent states that has been found to possess prominent quantum properties is well-known, and several suggestions have been proposed to generate such a coherent-state superposition and quantum entanglement for quantum information processing using the weak Kerr nonlinearity [1,2]. However, decoherence effects may be inevitable during the process of generating coherent-state superpositions because of the presence of nonlinear Kerr media. Recently, a displaced thermal state, which was defined as a pure coherent state $|\alpha\rangle$ with amplitude α subjected to a Gaussian noise

$$\mathbb{P}(V,d;\alpha)=\frac{2}{\pi(V-1)}e^{-\frac{2|\alpha-d|^2}{V-1}}, \tag{1}$$

was introduced [3–5], and its explicit integral form is described as

$$\rho(V,d)=\int d^2\alpha\,\mathbb{P}(V,d;\alpha)|\alpha\rangle\langle\alpha|, \tag{2}$$

where V is the thermal noise variance since the variance V changes with the temperature T of the thermal field according to the relation $e^{\hbar\omega/T}=(V+1)/(V-1)$, where ω is the frequency of thermal field, $\hbar$ is Planck's constant, and d is the displacement for the thermal field in phase space. For such a mixed superposed state, Jeong and co-workers have done a great deal of research and have obtained a series of high-quality symbolized achievements [3–5]. For instance, in Ref. [3] the transfer of nonclassicality from thermal-state superpositions to thermal states at the high-temperature limit has been realized based on the Wigner functions (WFs) for these superposed states. In Ref. [6], the decoherence problem of thermal-state superpositions in the photon-loss cavity (i.e., amplitude-decay mode) was studied and compared with that of coherent-state superpositions via the time-evolution of the negative minimum of the partial negative WF with the decoherence time-scales in phase space. However, to our knowledge, the analytical and numerical study of the probability distributions and the negative volumes of the

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WFs with the change of the parameters d and V has not been previously reported. Moreover, the negative volume should be a better candidate than the negative minimum of the partial negativity of the WF for quantifying the nonclassicality of any non-classical state according to the conclusions in Refs. [7–9].

Differing from the approaches of previous workers [3–6], in this paper we will analytically and numerically investigate the nonclassicality of thermal-state superpositions and their decoherence properties for amplitude decay recurring to the probability distributions and the negative volumes of the WFs with the parameters d and V. We present normal product representations of thermal-state superpositions and use it to derive some analytical results, which make for the numerical study of their nonclassicality and WF distributions, via the integration technique within normal products of operators. In addition, we derive the time-evolution of thermal-state superpositions for amplitude decay via the Kraus operator-sum representations of the density operators, and study their decoherence features by numerically analyzing the time-evolution WF.

2. Normal product representations of thermal-state superpositions

To analytically study a thermal-state superposition, we need to convert the density operator $\rho(V,d)$ in (2) into its normal product. Employing the integration technique within normal products of operators [10–13] and the operator identity $|0\rangle\langle 0| =: e^{-a^\dagger a}:$ [14], as well as the mathematical integral formula [15],

$$\int \frac{d^2\alpha}{\pi}\exp(-h|\alpha|^2 + s\alpha + g\alpha^*) = \frac{1}{h}\exp\left(\frac{sg}{h}\right), \tag{3}$$

to carry out the integration $\rho(V,d)$, we have

$$\rho(V,d) = \frac{2e^{-2|d|^2/(V+1)}}{V+1} : \exp\left[\frac{2}{V+1}(da^\dagger + d^*a - a^\dagger a)\right]:, \tag{4}$$

where the symbol $::$ represents normal ordering of the operators. Actually, such a state represents the intermediate Gaussian state between a mixed thermal state $\rho(V,0)$ with noise V and a pure coherent state $\rho(1,d)$ with amplitude d. In another respect, $V \to 1$ leads to $T \to 0$, which also tells us that $\rho(V,d)$ can be considered a generalization of pure coherent states to high-temperature thermal mixtures.

To generate a thermal-state superposition, one takes the displaced thermal states $\rho(V,d)$ for component states to interact with a superposition state $(|0\rangle_1 + |1\rangle_1)/\sqrt{2}$ ($|0\rangle$ and $|1\rangle$ being the ground and excited state, respectively, of a two-level system) in the cross-Kerr nonlinear media with nonlinear strength λ and the interaction Hamiltonian $\mathcal{H} = \lambda a^\dagger a b^\dagger b$ ($a^\dagger, b^\dagger$ represent the creation operators of modes 1 and 2). The output states are obtained as follows [3]:

$$\rho_{out}(V,d) = \frac{1}{2}\int d^2\alpha \mathbb{P}(V,d;\alpha)\left[|0,\alpha\rangle\langle 0,\alpha| + |1,\alpha e^{i\phi}\rangle\langle 0,\alpha| + |0,\alpha\rangle\langle 1,\alpha e^{i\phi}| + |1,\alpha e^{i\phi}\rangle\langle 1,\alpha e^{i\phi}|\right], \tag{5}$$

where $\phi = \lambda t$ is the phase angle related to the interaction time t. When one measures mode 1 on a superposed state $(|0\rangle_1 + |1\rangle_1)/\sqrt{2}$ and takes $\phi = \pi$ in the remaining states by controlling the interaction time t, the resultant states are just thermal-state superpositions with remarkable nonclassical properties, which can be written in normalized form as

$$\rho_\pm(V,d) = \mathcal{D}[\rho(V,d) + \rho(V,-d) \pm \sigma(V,d)], \tag{6}$$

where $\mathcal{D} = (2 \pm 2e^{-2|d|^2/V}/V)^{-1}$ is the normalization factor and the coherence term $\sigma(V,d)$ is given as $\sigma(V,d) = \int d^2\alpha \mathbb{P}(V,d;\alpha)|\alpha\rangle\langle-\alpha| + H.c.$. Likewise, we have

$$\begin{aligned}&\int d^2\alpha \mathbb{P}(V,d;\alpha)|\alpha\rangle\langle-\alpha| \\ &= \frac{2e^{-2|d|^2/(V+1)}}{V+1} : \exp\left[\frac{2}{V+1}(da^\dagger - d^*a - Va^\dagger a)\right]: \\ &\equiv \rho'(V,d)\end{aligned} \tag{7}$$

and $H.c. \equiv \rho'(V,-d)$. Note, however, that $\rho_\pm(V,d)$ becomes a classical mixture of two local Gaussian states when $\sigma(V,d) = 0$; thus we say that the coherence term $\sigma(V,d)$ is fully responsible for the nonclassical behaviors of thermal-state superpositions. Noting that the sum of two normal ordered products still remains normal ordered, i.e., $:W: + :V: =: W + V:$, the normal product representations of thermal-state superpositions $\rho_\pm(V,d)$ are thus

$$\begin{aligned}\rho_\pm(V,d) = \frac{4\mathcal{D}e^{-2|d|^2/(V+1)}}{V+1} : \Bigg[&e^{-\frac{2a^\dagger a}{V+1}}\cosh\frac{2(d^*a + da^\dagger)}{V+1} \\ &e^{-\frac{2Va^\dagger a}{V+1}}\cosh\frac{2(d^*a - da^\dagger)}{V+1}\Bigg]:.\end{aligned} \tag{8}$$

For $V = 1$, $\rho_\pm(V,d)$ reduces to a superposition of two pure coherent states $|\pm d\rangle$, i.e., $\rho_\pm(1,d) \propto |\phi\rangle\langle\phi|$, where $|\phi\rangle = |d\rangle \pm |-d\rangle$. For a physical realization, several important experiments have already demonstrated an ideal coherent-state superposition $\rho_\pm(1,d)$ with small amplitude d [16,17]. Whereas $d = 0$, $\rho_\pm(V,0) \propto e^{a^\dagger a\ln\left(\frac{V-1}{V+1}\right)} \pm e^{a^\dagger a\ln\left(\frac{1-V}{V+1}\right)}$, a superposed thermal field. Also, $\rho_\pm(V,0)$ can be rewritten as

$$\rho_\pm(V,0) \propto \begin{cases}[1 \pm (-1)^{a^\dagger a}]e^{a^\dagger a\ln\left(\frac{V-1}{V+1}\right)}, & V > 1 \\ [(-1)^{a^\dagger a} \pm 1]e^{a^\dagger a\ln\left(\frac{1-V}{V+1}\right)}, & V < 1\end{cases}, \tag{9}$$

where $(-1)^{a^\dagger a}$ is the parity operator, whose eigenvectors are number states $|n\rangle$ with the corresponding eigenvalues $(-1)^n$. In addition, it is worth pointing out that the normal product of $\rho_\pm(V,d)$ makes it convenient to investigate nonclassical effects, WF distributions and decoherence characteristics.

3. Observable nonclassical effects

In this section, we investigate two observable nonclassical effects, quadrature squeezing and sub-Poissonian distribution, resulting from the coherence term $\sigma(V,d)$.

Note that the squeezing of a quadrature operator $X_\theta = ae^{-i\theta} + a^\dagger e^{i\theta}$ is characterized by the normal ordering inequality $\langle:(\triangle X_\theta)^2:\rangle < 0$. Upon calculating the expectation value $\langle:(\triangle X_\theta)^2:\rangle$, one can define the squeezing degree S as

$$S = -2\left|\langle a^{\dagger 2}\rangle - \langle a^\dagger\rangle^2\right| + 2\langle a^\dagger a\rangle - 2|\langle a^\dagger\rangle|^2, \tag{10}$$

where S always holds for all angles θ, and its value range $[-1,0)$ signifies quadrature squeezing. Under the states $\rho_\pm(V,d)$, the expectation values $\langle a^\dagger\rangle = 0$, $\langle a^{\dagger 2}\rangle = 2\mathcal{D}d^2(V^3 \pm e^{-2|d|^2/V})/V^3$, and $\langle a^\dagger a\rangle = \mathcal{D}\{V^3(2|d|^2 + V - 1) \mp [2|d|^2 + V(V-1)]e^{-2|d|^2/V}\}/V^3$; thus the degree of squeezing $S_+(V,d)$ of $\rho_+(V,d)$ is directly obtained from the results above, as shown in the plot of $S_+(V,d)$ as a function of d (here and hereafter, d is taken as real without loss of generality) and V in Fig. 1(a).

From Fig. 1(a), one can clearly see that the state $\rho_+(V,d)$ can exhibit squeezing in a symmetric value region of d in the vicinity of $V = 1$, and the symmetric range of d becomes small with the increase of V. **Especially, when $d = 0$, $\rho_+(V,d)$ cannot show non-zero squeezing for the vicinity of $V = 1$ owing to $S_+(V,d) \geqslant 0$.** However, the state $\rho_-(V,d)$ never produces squeezing for any value of V and d. In addition, the squeezing degree $S_+(V,d)$ increases and then decreases with d for a fixed value of V, but $S_+(V,d)$ decreases with increasing V for a given d.

Next, we investigate the sub-Poissonian features in order to characterize the nonclassicality of $\rho_\pm(V,d)$ by using the Mandel's Q-factor represented by $Q_\pm = (\langle a^{\dagger 2}a^2\rangle - \langle a^\dagger a\rangle^2)/\langle a^\dagger a\rangle$, whose negative values in the range $[-1,0)$ refer to the sub-Poissonian distribution of this state. Noting the expectation values $\langle a^\dagger a\rangle$ and $\langle a^{\dagger 2}a^2\rangle = \mathcal{D}\{V^5[(2|d|^2 + V - 1)^2 - 2|d|^4] \pm [(2|d|^2 + V^2 - V)^2 - 2|d|^4]e^{-2|d|^2/V}\}/V^5$, we find that the Mandel's factor Q_- can always take some negative values in the range $[-1,0)$ for certain d and V, and the negativity of Q_- in this case increases and then decreases with d, but always decreases with V, as shown in Fig. 1(b). However, $\rho_+(V,d)$ does not yield a sub-Poissonian distribution at all.