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Stability of two-degrees-of-freedom aero-elastic models with frequency and time variable parametric self-induced forces

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ABSTRACT

The lowest critical state of slender systems representing long suspension bridges can be investigated using two degree of freedom linear models. Initially, the neutral model with aero-elastic forces treated as constants can be used and such approach works well on the theoretical level. However, because time dependency is neglected, it is naturally limited to the very close neighbourhood of the bifurcation point. Thus, an approach using aero-elastic coefficients known as flutter derivatives was introduced in the past. The present paper combines these models together on one common basis and establishes linkage to avoid the time–frequency duality. The stability limits are analysed by means of the generalized Routh–Hurwitz approach and Liénard theorems. Some examples of bridge stability analyses are provided using experimentally ascertained or literature based data.

1. Introduction

Dynamic wind effects on long bridges are frequently analysed using a two degrees of freedom (2DOF) model working with heave and torsional response components of a representative cross-section. The wind–structure interaction based on the energy extraction from the flow leads to the formation of the non-conservative and gyroscopic self-excited forces and to possible loss of the dynamics stability. There are several types of self-excited vibration, with different characteristics from both the applied mathematics and the engineering points of view, see e.g. Borri and Höffer (2000), Scanlan (2001), Bartoli and Righi (2006) and Matsumoto et al. (2005). They can be observed separately depending on the geometry and the mechanical properties of a structure and on the wind flow conditions. Very often, however, the distinction may not be entirely evident, especially because variations in the interaction of these oscillation types can appear. Relevant mathematical models in the literature differ in principle in terms of their composition of aero-elastic forces. This enables a rough classification into three groups.

The first group can be possibly called neutral models, as it is used in Náprstek and Pospíšil (2012). Coefficients in relevant differential equations are introduced as suitable parameters that consist (at the linear level) of structural parts and of the corresponding aero-elastic contributions, which are the functions of the mean velocity. In contrast, the other groups seek the solution of the aero-elastic equations in the non-autonomous form, taking the time and the frequency (and wind velocity) into the consideration.

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Essentially, the second group uses aero-elastic coefficients named as direct and indirect flutter derivatives (FD), see e.g. Brownjohn and Bogunovic-Jakobsen (2001) or Chen and Kareem (2003) and solves the "combined time-frequency" system by an iteration directly determining the critical velocity and the frequency of the resulting coupled heave-pitch motion. Several other solutions have been proposed by different authors. A frequency-domain approach based on FD and complex eigenvalue analysis and its equivalent time-domain counterpart and direct integration of the governing equations is discussed in Salvatori and Borri (2007). Similar inconsistency in formulation between frequency-dependent aeroelastic terms and quasi-static terms has been solved in Scanlan and Jones (1999). In Šarkić et al. (2012), the results of numerical investigations of static coefficients and non-stationary FD for a symmetric bridge deck section are presented. Some of the numerical techniques were presented in Robertson et al. (2003), where a fluid solver has been modified to incorporate a body undergoing translational and rotational motion. A rather novel frequency-independent approximation of the self-excited forces is presented in Øiseth et al. (2011), which for the suspension bridge considered provides results as accurate as those from the unsteady models. The same authors made a successful attempt for an analytical solution of the flutter problem, see Øiseth and Sigbjörnsson (2011). A third degree algebraic equation for the critical velocity is presented there.

The third group works with the so-called indicial functions (IF); these are defined as kernels of convolution integrals formulating aero-elastic forces as functions of time, see Wagner (1929), Garrick (1938) and Scanlan et al. (1974). The second and the third group can be transformed, so that they can be mutually related and reflect the duality of time and frequency domains, see Caracoglia and Jones (2003a, 2003b), Costa (2007), Costa et al. (2007) and Neuhaus et al. (2009). Each of these formulations affords some advantages. The general time-domain description of the loads using indicial functions and the extension of the framework of thin air-foil theory to mildly bluff sections, has been presented recently in de Miranda et al. (2013).

Despite being applied successfully in practice, from mathematical point of view, many of these procedures are considerably problematic as they imply several assumptions that probably will not be complied with as soon as the response loses the character of a purely harmonic movement with a clearly expressed frequency. Another shortcoming of these approaches consists in orientation on one combination of parameters. Therefore, parametric study requires a number of individual evaluations and subsequently the assessment of the relevant data set.

The aim of this paper is to compose a new formulation which enables a qualitatively more general insight into the stability loss mechanism considering all involved parameters and to present analytical and thus physically instructive closed form. To reach these goals, the paper combines the main aspects of the first (neutral) and second groups (flutter derivatives), thereby joining these models together on one common basis and establishing linkages that avoid the time–frequency duality.

In principle, for an analytical investigation, the indicial functions could be used as an adequate basis too. Nevertheless, there are significant differences in applications regarding the numerical stability and the result resolution. The authors decided for the FD approach mainly because

- FD are primary results obtained in aero-dynamic tunnel, while indicial functions should be subsequently evaluated by complicated filtering and transformations.
- The variant with flutter derivatives is conducted in the frequency domain which is a process much more numerically stable in comparison with operation in the time domain; it applies especially to fitting of model parameters using experimental data.
- Kernels of relevant integrals using indicial functions are exponential and this case is very often a dangerous resource of global numerical instability (unless the optimal filtering is used), especially in the vicinity of the bifurcation point, due to combination of the perturbed multiplicative coefficients and the coefficients in exponents.
- Analysis of system properties in the frequency domain is easier and more flexible when the multi-parameter system character is investigated, while solution in the time domain should be preferred when time isolated singularities are decisive. For instance, stability conditions analysis as functions of all system parameters and one bifurcation parameter is possible only in frequency domain. Stability due to one isolated impact can be properly assessed in time domain using, e.g., Lyapunov exponent in numerical simulation for one particular parameter configuration.

The stability limits themselves are analysed by means of the generalized Routh–Hurwitz approach completed by the Descartes and Liènard theorems. The method of investigation, which is outlined here, uses conventional procedures of the mathematical analysis. It is an extension of an approach described in detail in Náprstek and Pospíšil (2012), including the phenomenological interpretation of the stability criteria for the linear model with respect to the elastic as well as the aerodynamic meaning of the term, where also physical conclusion were found regarding the flutter and divergence, vibration regimes and type of the flutter e.g. by identification of the phase shift between individual components. At the end of this paper, practical application of the method is presented.

For the purposes of this study, a girder in Fig. 1 of mass m [kg/m] and mass moment of inertia I [kg m] per one meter is considered. In effect, it has an axially symmetric cross-section with possible response components in the heave u (vertical direction) and pitch φ (rotation around S point).