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Parameter estimation in plasmonic QED

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A B S T R A C T

We address the problem of parameter estimation in the presence of plasmonic modes manipulating emitted light via the localized surface plasmons in a plasmonic waveguide at the nanoscale. The emitter that we discuss is the nitrogen vacancy centre (NVC) in diamond modelled as a qubit. Our goal is to estimate the β factor measuring the fraction of emitted energy captured by waveguide surface plasmons. The best strategy to obtain the most accurate estimation of the parameter, in terms of the initial state of the probes and different control parameters, is investigated. In particular, for two-qubit estimation, it is found although we may achieve the best estimation at initial instants by using the maximally entangled initial states, at long times, the optimal estimation occurs when the initial state of the probes is a product one. We also find that decreasing the interqubit distance or increasing the propagation length of the plasmons improve the precision of the estimation. Moreover, decrease of spontaneous emission rate of the NVCs retards the quantum Fisher information (QFI) reduction and therefore the vanishing of the QFI, measuring the precision of the estimation, is delayed. In addition, if the phase parameter of the initial state of the two NVCs is equal to π rad, the best estimation with the two-qubit system is achieved when initially the NVCs are maximally entangled. Besides, the one-qubit estimation has been also analysed in detail. Especially, we show that, using a two-qubit probe, at any arbitrary time, enhances considerably the precision of estimation in comparison with one-qubit estimation.

1. Introduction

The field of cavity quantum electrodynamics (cavity QED) arised to investigate and control the light–matter interaction, specially the coupling of two-level emitters [1], such as quantum dots or superconducting qubits, to any bosonic monomode quantized in, e.g., superconducting cavities [2], carbon nanotubes [3], nanomechanical resonators [4], (collective) spin waves in molecular crystals [2], or photonic cavities [5]. On the other hand, plasmonic QED [6] is another interesting field of research focusing on new opportunities for studying and controlling the interaction of light with matter in the presence of plasmonic modes manipulating light via the localized surface plasmons. The advanced fabrication techniques make the plasmonic nanostructure an important candidate for quantum optics, quantum information processing (QIP) and quantum control, e.g., atomic spectroscopy [7], single-photon transistors [8], superradiance [9], focusing [10], lasing [11], and single-plasmon emission [12].

The emitter we consider here is the NVC in diamond. The NVC consists of a substitutional nitrogen atom connected with a vacancy in the diamond crystal lattice. Under suitable conditions, a NVC forms an effective two-level system. In fact, an individual NVC may be viewed as a fundamental unit of a quantum computer in which the centre electron spin with a very high control speed acts as the probe and the nuclear spin with a long coherence time performs as the memory. This interesting solid-state system is one of the most important candidates for QIP, and many manipulation processes and coherent control have been performed with this system [13–21]. In fact, the necessary operations for quantum computing, including initialization [22], the manipulation [23], storage [24], readout [16], and significant interactions between two diamond NV spins [25,26], have been well demonstrated experimentally. Besides, there are a number of other important demonstrations, such as the quantum controlled-ROT gate [27], Deutsch–Jozsa algorithm [28], holonomic CNOT gate [29], and decoherence-protected controlled-rotation gates [17]. In addition, based on photon–NV entangled platform [30], Bell inequality and quantum teleportation between distant diamond NVCs have been experimentally demonstrated [26,31], and a number of proposals for implementing parallel and hyper-parallel quantum computing have been also proposed [32,33].

Confining the electromagnetic field in a plasmonic waveguide (PW), well below the diffraction limit, the plasmons lead to strong local fields around emitters and may be guided along the interface in the form of a travelling wave known as a surface plasmon–polariton (SPP) [34]. More precisely, SPPs are surface wave quanta bound to the interface, usually

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separating a dielectric and a metal. Their quasi-particle nature originates from the interaction of light waves with the free electrons of the metal. Such resonant response of the electrons leads to collective oscillations called plasma oscillations, which trap the light at the surface of the metal. The light field together with those plasma oscillations, constitute the SPPs. In fact, surface plasmon waves, revealing the macroscopic motion of SPPs, are a coherent combination of plasmons and photons.

Because the information about the world is obtained by observation and measurement, the results are subject to error. The approach proposed by classical estimation theory for reducing the statistical error, is to increase the number of resources for the measurement in accordance with the central limit theorem; however, this method is sometimes inefficient and undesirable [35]. On the other hand, quantum parameter estimation [36], the emerging field of quantum technology, intends to yield higher statistical precision of an unknown parameter by using and controlling quantum resources than purely classical approaches [37]. There have been different studies on precision of parameter estimation in many physical systems, such as optical interferometry [35,38], atomic systems [39,40], and Bose–Einstein condensates [41]. Nevertheless, to the best of our knowledge, only few papers have been devoted to investigate the parameter estimation in quantum systems composed of NVCs. In a recent paper [42], the authors reported the room-temperature proof of principle implementation of entanglement-enhanced phase estimation in the NVC in pure diamond single crystal. Moreover, in Ref. [43], the quantum estimation theory is used to optimize the number of pulses applied to NVCs in order to determine the amplitude, phase, and frequency of unknown weak magnetic fields.

In this paper, we investigate the process of parameter estimation in a system composed of two separated NVCs, playing the role of emitters, placed near a one-dimensional plasmonic waveguide (PW). When these emitters are placed close to a metal surface, they couple to nonpropagating, quickly decaying modes and hence the energy is dissipated through heating of the metal. The rate at which the energy is dissipated may be very high when the emitters are close to a metal surface. In fact, this rate is the dominant decay rate if the emitter–metal distances become below 10 nm. To avoid this quenching and suppressing effect, one may lift the NVCs away from the metal surface and place it at an intermediate region close enough in order to couple efficiently to plasmons. Through the paper, we assume that the NVCs are placed at a distance of 10 nm from the metal surface. This can be verified by estimation of the fraction of energy, radiated by the emitters, and also captured by waveguide surface plasmons (the so-called "β factor") [44]. Motivated by this, we address the estimation of parameter β determining the degree of efficient coupling of NVCs to propagating plasmonic modes. By calculating the QFI, we analysed how the precision of the parameter estimation may be affected by initial conditions and different control parameters, such as spontaneous emission rate, the distance between the two NVCs, and propagation length of the plasmon. We also analyse how increasing the number of qubits enhances the parameter estimation.

This paper is organized as follows. In Section 2, the model and the master equation are introduced. In Section 3, a brief review of the QFI and its relation to Uhlmann fidelity are presented. In Section 4, the problem of β estimation by using NVCs as probes, is discussed completely. Moreover, in Section 5 we analyse the difference between one-qubit and two-qubit probing. Finally, Section 6 is devoted to the conclusion.

2. The model

We consider two separated NVCs (NVC1 and NVC2) coupled to the modes supported by a 1D PW, as shown in Fig. 1. Each NVC, negatively charged with two unpaired electrons located near the vacancy, may be modelled as a qubit with the transition frequency ω_0. Tracing out

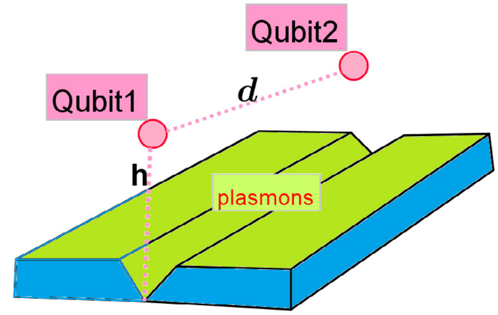

Fig. 1. The composite NVC–PW system consisting of two identical NVCs in diamond nanocrystals and a 1D plasmonic waveguide.

over the PW degrees of freedom and employing the Born–Markovian approximation, we can obtain the master equation for two NVCs [6,45]:

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{i,j=1,2} \frac{\Gamma_{ij}}{2} \left[2\sigma_i^- \rho(t)\sigma_j^+ - \sigma_j^+\sigma_i^-\rho(t) - \rho(t)\sigma_j^+\sigma_i^-\right], \tag{1}$$

where the Hamiltonian is given by

$$H = \sum_{j=1,2} \left[(\omega_0 + g_{jj})\sigma_j^+\sigma_j^- + g_{12}(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)\right], \tag{2}$$

in which $\sigma_i^+ = |e\rangle\langle g|$ and $\sigma_i^- = (\sigma_i^+)^\dagger$ represent the raising and lowering operators for qubit i. Moreover, Γ_{jj} and g_{jj} are the individual spontaneous emission rate and frequency shift to each NVCs, respectively. In addition, Γ_{12} and Γ_{21} ($\Gamma_{12} = \Gamma_{21}$) represent respectively the correlated spontaneous emission rate and the effective coupling strength of the two qubits modulated by the PW. It is possible to extract both g_{ij} and Γ_{ij} from knowing the dipole moments and the classical Green's tensor in the presence of the PW.

Assuming that the qubits are placed at equivalent positions along the PW, we label $\Gamma \equiv \Gamma_{ii}$. On the other hand, for distances larger than about h = 10 nm, g_{jj} is very small at optical frequencies, and hence it is neglected in this paper.

When in the PW the β-factor, measuring the fraction of the emitted radiation captured by the propagating mode, is high, the effective coupling strength g_{12} and cross-decay rate Γ_{12} may be expressed as [45]:

$$g_{12} = \frac{\Gamma}{2}\beta \sin(k_{pl}d)e^{-d/(2l)}; \quad \Gamma_{12} = \Gamma\beta\cos(k_{pl}d)e^{-d/(2l)} \tag{3}$$

where d is the interqubit distance in the nanometre range. Moreover, l and $k_{pl} = 2\pi/\lambda_{pl}$ ($\lambda_{pl} = 637$ nm in our case) represent the propagation length and wave-number of the plasmon, respectively.

Preparing the two NVCs in the Bell-like states $|\Phi\rangle = \sqrt{\alpha}|10\rangle + e^{i\theta}\sqrt{1-\alpha}|01\rangle$ and solving master equation (2), one finds that the non-zero elements of evolved density matrix $\rho(t)$ are given by [6]:

$$\begin{aligned}
\rho_{00,00} &= 1 - e^{-\Gamma t}\left[\cosh(\Gamma_{12}t) - 2\xi\cos\theta\sinh(\Gamma_{12}t)\right],\\
\rho_{01,01} &= \frac{1}{2}e^{-\Gamma t}\left[(2\alpha-1)\cos(2g_{12}t) + \cosh(\Gamma_{12}t)\right.\\
&\quad\left. + 2\xi\sin\theta\sin(2g_{12}t) - 2\xi\cos\theta\sinh(\Gamma_{12}t)\right],\\
\rho_{01,10} &= \frac{1}{2}e^{-\Gamma t}\left[i(2\alpha-1)\sin(2g_{12}t) - \sinh(\Gamma_{12}t)\right.\\
&\quad\left. - 2i\xi\sin\theta\cos(2g_{12}t) + 2\xi\cos\theta\cosh(\Gamma_{12}t)\right],\\
\rho_{10,10} &= 1 - \rho_{00,00} - \rho_{01,01}
\end{aligned} \tag{4}$$

where $\rho_{mn,m'n'} = \langle mn|\rho(t)|m'n'\rangle$ and $\xi = \sqrt{\alpha(1-\alpha)}$.

3. Quantum metrology and Uhlmann fidelity

When states ρ_θ and $\rho_{\theta+d\theta}$ differing from each other by an infinitesimal change $d\theta$, parameter θ may be estimated with high accuracy.