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Controlling orbital angular momentum of an optical vortex by varying its ellipticity

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A B S T R A C T

An exact analytical expression is obtained for the orbital angular momentum (OAM) of a Gaussian optical vortex with a different degree of ellipticity. The OAM turned out to be proportional to the ratio of two Legendre polynomials of adjoining orders. It is shown that if an elliptical optical vortex is embedded into the center of the waist of a circularly symmetrical Gaussian beam, then the normalized OAM of such laser beam is fractional and it does not exceed the topological charge n. If, on the contrary, a circularly symmetrical optical vortex is embedded into the center of the waist of an elliptical Gaussian beam, then the OAM is equal to n. If the optical vortex and the Gaussian beam have the same (or matched) ellipticity degree, then the OAM of the laser beam is greater than n. Continuous varying of the OAM of a laser beam by varying its ellipticity degree can be used in optical trapping for accelerated motion of microscopic particles along an elliptical trajectory as well as in quantum informatics for detecting OAM-entangled photons.

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1. Introduction

Optical vortices without circular symmetry and with a fractional orbital angular momentum (OAM) are intensively studied in the last few years. This is due to applicability of the fractional-OAM optical vortices in quantum informatics to detect the OAM-entangled photons [1–3]. At the same time, maximal degree of quantum link security is achieved.

There are several ways to generate a fractional-OAM optical vortex. For example, it can be done by shifting the Gaussian beam center from the center of the spiral phase plate [4,5], or by using a non-integer 2π phase step spiral phase plate [6,7], or by superposition of light modes with different values of integer topological charge [8,9]. Similar expansion of the fractional-OAM Bessel beams has also been studied theoretically in [10] and experimentally in [11]. Vortices with the fractional OAM can also be obtained by generating asymmetric optical vortices with the crescent-shape intensity distributions [12,13]. However, in this case the trajectory of microparticles motion will be split. So, it is interesting to study the case when the OAM is fractional and the light intensity curve is closed in the transverse plane. The easiest way to do this is to generate an elliptical optical vortex. In [14], the transformation of an optical vortex is investigated by embedding into it a different degree of ellipticity. The work [14] continues earlier works on the study of elliptical optical vortices [15,16]. However, the OAM of elliptic vortices was not investigated in [14–16]. The closest to this work is the paper [17]. In [17], an elliptical vortex Hermite-Gaussian beam was considered and its OAM was calculated, which turned out to be fractional. However, in order to generate such a beam in practice, it is necessary to use an elliptical Gaussian beam as the incident onto the amplitude-phase optical element, but combining the given amplitude and phase in one element is a difficult task.

There exist also other ways of controlling the angular momentum of a laser beam [18,19]. In [19], complex vector light fields are considered. A photon in such field has a unit spin at points with the circular polarization, while at points with the elliptical polarization it has a fractional spin, which is less than unity. In [18], an elliptic Gaussian beam is considered, which is focused by a cylindrical lens. The OAM of such a beam varies depending on the degree of the Gaussian beam ellipticity and on the inclination angle of the cylindrical lens.

In this work, a simple formula is obtained for the OAM of an elliptic Gaussian vortex with an n-fold degenerate intensity null on the optical axis (in the Gaussian beam center). It follows from this formula that depending on the type of the optical vortex ellipticity, the increasing of ellipticity can make the OAM to increase or decrease, or remain

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unchanged. It turns out that if an elliptical vortex is embedded into the center of a circularly symmetric Gaussian beam, then the OAM of the whole beam, normalized to its energy, is fractional and it does not exceed the vortex topological charge. In addition, the OAM decreases with increasing of the vortex ellipticity. If both the Gaussian beam and the optical vortex have a matched (similar or equal) ellipticity, then the normalized OAM of the whole beam exceeds its topological charge, and it increases with the increasing ellipticity. We already studied an elliptic optical vortex embedded into a Gaussian beam [20], but in [20] we considered only a circular Gaussian beam. Here we study more general case when both the vortex and the Gaussian beam are elliptic (with different ellipticity).

2. Orbital angular momentum of an elliptic Gaussian beam with an embedded intensity null

We consider an isolated n-fold degenerate elliptic intensity null embedded in the center of the waist of an elliptic Gaussian beam:

$$E_n(x,y)=\left(\frac{x}{a_x}+i\frac{y}{a_y}\right)^n\exp\left(-\frac{x^2}{2w_x^2}-\frac{y^2}{2w_y^2}\right), \tag{1}$$

where n is the integer topological charge of the optical vortex, a_x and a_y are positive values, which determine the ellipticity of the optical vortex (n-fold degenerate isolated intensity null), w and w_y are the waist radii of the elliptical Gaussian beam along the Cartesian axes. The orbital angular momentum (OAM) and the beam power (energy) of the paraxial field (1) are determined by the well known equations [5,12,13]:

$$J_z=\operatorname{Im}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{E}_n(x,y)\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_n(x,y)\,dx\,dy, \tag{2}$$

$$W=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{E}_n(x,y)\,E_n(x,y)\,dx\,dy, \tag{3}$$

where Im means imaginary part of a complex number, $\bar{E}$ is the conjugated complex amplitude. Then, instead of (2) and (3) we get for the field (1):

$$\frac{J_z}{W}=n\left[a_xa_y\frac{w_y^2-w_x^2}{a_x^2w_y^2-a_y^2w_x^2}+w_xw_y\frac{a_x^2-a_y^2}{a_x^2w_y^2-a_y^2w_x^2}\frac{P_{n-1}(x)}{P_n(x)}\right], \tag{4}$$

where $x=\left[\left(w_xw_y\right)/\left(2a_xa_y\right)\right]\left[\left(a_x/w_x\right)^2+\left(a_y/w_y\right)^2\right]$ and $P_n(x)$ is the Legendre polynomial [21]. From Eq. (4), interesting partial cases follow, which show how one can control the OAM of a laser beam in a wide range by using the beam ellipticity.

3. Partial cases

1. If the optical vortex (1) is circularly symmetric ($a_x=a_y$), while the Gaussian beam has an elliptical waist ($w_x\neq w_y$), then Eq. (4) leads to:

$$J_z/W=n. \tag{5}$$

This means that the OAM of a circularly symmetric optical vortex, embedded into the center of the waist of an elliptical Gaussian beam, equals the topological charge. So, even if the shape of the real amplitude function (1) depends on the azimuth angle φ, it does not affect the OAM. The OAM value is affected only by the phase component of the beam complex amplitude (1), which in this case is represented by single angular harmonic $\exp(in\varphi)$, where φ is the azimuthal angle in the polar coordinates (r,φ).

2. If an elliptical optical vortex ($a_x\neq a_y$) is embedded into the center of the waist of a circularly symmetric Gaussian beam ($w_x=w_y=w$), then instead of Eq. (4) we get:

$$J_z/W=nP_{n-1}(y)\left[P_n(y)\right]^{-1},\quad y=\left(a_x^2+a_y^2\right)\left(2a_xa_y\right)^{-1}. \tag{6}$$

From Eq. (6) it follows that if only vortex part of the beam is elliptical then the OAM is fractional and it is less than the topological charge n. Indeed, since $a_x^2+a_y^2\geq 2a_xa_y$, then $y\geq 1$, while it is known [21] that $P_{n-1}(y)\leq P_n(y)$ at arbitrary $y\geq 1$ for any order $n>0$. The equality is achieved at $a_x=a_y$ (i.e. for circularly symmetric optical vortex), since $P_n(1)=1$ for any $n>0$. In order to explain, why the OAM (6) is less than n, we transform the amplitude (1) by using the Newton binomial:

$$E_n(r,\varphi)=\left[\frac{r\left(a_x+a_y\right)}{2a_xa_y}\right]^n\exp\left(-\frac{r^2}{2w^2}\right)\times\sum_{k=0}^{n}\frac{n!}{k!\,(n-k)!}\left(\frac{a_y-a_x}{a_y+a_x}\right)^k e^{i(n-2k)\varphi}. \tag{7}$$

It is seen in Eq. (7) that $2n$ different-weight angular harmonics $\exp(\pm ik\varphi)$ contribute to the field amplitude. It can be shown, that the OAM of the sum of n angular harmonics with weights, normalized by the full energy, is less than the OAM of one angular harmonic with the maximal number n. Indeed, let us consider a light field with the complex amplitude, consisting of sum of a finite number of the angular harmonics: $F(r,\varphi)=\sum_{k=0}^{k=n}A_k(r)\exp(ik\varphi)$. According to Eqs. (2) and (3), the normalized OAM of such field reads as $J_z/W=\sum_{k=0}^{k=n}k\widetilde{I}_k$, where $I_k=\int_0^\infty\left|A_k(r)\right|^2rdr$ and $\widetilde{I}_k=I_k\left(\sum_{k=0}^{k=n}I_k\right)^{-1}$. This leads to an obvious inequality: $J_z/W=\sum_{k=0}^{k=n}k\widetilde{I}_k\leq n\sum_{k=0}^{k=n}\widetilde{I}_k=n$.
Thus, we have shown that an elliptical vortex, embedded in the center of the waist of a circularly symmetric Gaussian beam, contains a finite number of angular harmonics, making the normalized OAM to be less than the topological charge. It also follows from Eq. (6) that the greater is the degree of ellipticity of the vortex, i.e. the larger the ratio a_x/a_y differs from unity, the smaller is the OAM of the light beam (1). It can be explained physically, since the OAM density of the elliptical vortex is greater at the elongated parts of the ellipse, which, at the same time, are furthest from the center of the Gaussian beam, and in which the light intensity is minimal. In addition, decreasing of the OAM of an elliptical vortex, embedded in the center of the waist of a circularly symmetric Gaussian beam, can be explained by analyzing the distribution of the OAM density normalized to the intensity:

$$j_z=\operatorname{Im}\left(\bar{E}_n(r,\varphi)\frac{\partial E_n(r,\varphi)}{\partial\varphi}\right),\quad I=\left|E_n(r,\varphi)\right|^2. \tag{8}$$

In this case, the normalized OAM density reads as

$$j_z/I=\frac{n}{a_xa_y}S^2(\varphi),\ I=r^{2n}\exp\left(-\frac{r^2}{w^2}\right)S^{-2n}(\varphi), \tag{9}$$

where $(r,\ \varphi)$ are the polar coordinates, $S(\varphi)=\left(a_x^{-2}\cos^2\varphi+a_y^{-2}\sin^2\varphi\right)^{-1/2}$ is the radius of vortex ellipse in the direction of the polar angle φ. It is seen in Eq. (9) that if $a_x>a_y$, then at $\varphi=0$ or $\varphi=\pi$ the normalized OAM density (9) is maximal and equals $j_z/I=a_x\,n/a_y$, while at $\varphi=\pi/2$ or $\varphi=3\pi/2$ the OAM density is minimal and equals $j_z/I=a_y\,n/a_x$. So, in the parts of the ellipse with the maximal curvature the normalized OAM density is also maximal and exceeds the topological charge, while in the parts with the minimal curvature the OAM density is also minimal and is less than n. However, the total OAM depends on the summary OAM density. Since the intensity distribution (9) $I\ (r,\varphi)$ depends on φ, we cannot integrate the OAM density j_z/I in Eq. (9) only over the azimuthal angle φ, and therefore the total normalized OAM for the optical vortex (1) at $w_x=w_y$ and $a_x\neq a_y$ is as follows:

$$\frac{J_z}{W}=\frac{n}{a_xa_y}\left[\int_0^{2\pi}S^{2(1-n)}(\varphi)\,d\varphi\right]\left[\int_0^{2\pi}S^{-2n}(\varphi)\,d\varphi\right]^{-1}\leq n. \tag{10}$$

Eq. (10) can be expressed via the ratio of the Legendre polynomials and coincides with Eq. (6). It can be derived from the following equation:

$$P_n\left(\frac{a_x^2+a_y^2}{2a_xa_y}\right)=\frac{\left(a_xa_y\right)^n}{2\pi}\int_0^{2\pi}S^{-2n}(\varphi)\,d\varphi, \tag{11}$$

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