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# Speckle phase near random surfaces

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ABSTRACT

Based on Kirchhoff approximation theory, the speckle phase near random surfaces with different roughness is numerically simulated. As expected, the properties of the speckle phase near the random surfaces are different from that in far field. In addition, as scattering distances and roughness increase, the average fluctuations of the speckle phase become larger. Unusually, the speckle phase is somewhat similar to the corresponding surface topography. We have performed experiments to verify the theoretical simulation results. Studies in this paper contribute to understanding the evolution of speckle phase near a random surface and provide a possible way to identify a random surface structure based on its speckle phase.



## 1. Introduction

When an incident light passes through a random diffuser, or is reflected by the diffuser, intensity fluctuations named speckle patterns are generated [1]. As we all know, speckle fields near the exit of the diffuser contain many information of the diffuser. For instance, the effective refractive index of a disordered dielectric structure determines near-field short-range correlation of its speckle intensity [2]. For a disordered photonic crystal with incomplete band gap, the stop-band width can be extracted from its speckle patterns [3]. Theoretical results suggest that the speckle intensities produced by two incoherent sources contain information about the relative distance between the two sources [4]. It is acquired experimentally and theoretically that the non-universal behavior of the near-field speckle patterns, produced by a disordered dielectric medium at sub-wavelength distances, is associated with the internal structure of the medium [5–8]. Most of the past researches lay emphasis on the connections between the statistical parameters of a diffuser and the statistical properties of its speckle intensity [9–14]. The information about speckle phase is usually ignored. In this paper, we focus attention on the relation between the topography of different glass surfaces and the phase of the speckle fields in vicinity of them. Simultaneously, the evolutions of this relation and the speckle phase statistical properties are also presented.

## 2. Numerical studies on speckle phase

### 2.1. Formula for light scattering

Fig. 1 shows a diagrammatic sketch for light scattering. A random surface, with height distribution $z = h(x_1, y_1)$ and $\vec{n}$ unit normal vector at random point of the surface, is located on object plane $x_1 y_1$. A beam of parallel laser light with wavelength $\lambda$ illuminates the surface and speckle fields are formed behind it. Based on Kirchhoff approximation theory, the light field $U(x, y)$ on observation plane can be written as

$$U\,(x, y) = 1/4\pi \iint (U_0 \partial G/\partial n - G \partial U_0/\partial n) \mathrm{d}x_1 \mathrm{d}y_1 \tag{1}$$

where, $U_0(x_1, y_1) = \exp[-jn_0 k_0 h(x_1, y_1)]$ is the wave field at point $(x_1, y_1)$ of the surface, and $G = \exp(-jk_0 r)/r$ is Green's function of optical system, with $n_0$ being the refractive index of the surface, $r = \left[(x-x_1)^2 + (y-y_1)^2 + (z-h)^2\right]^{1/2}$ a distance between observation point $(x, y, z)$ and object point $(x_1, y_1, h)$ and $k_0 = 2\pi/\lambda$ amplitude of wave vector. $\partial/\partial n$ in Eq. (1) refers to derivative with respect to $\vec{n}$, and it is given by

$$\partial/\partial n = \vec{n} \cdot \nabla = 1/A((-\partial h/\partial x_1)(\partial/\partial x_1) - \left(\partial h/\partial y_1\right)(\partial/\partial y_1) + \partial/\partial z) \tag{2}$$

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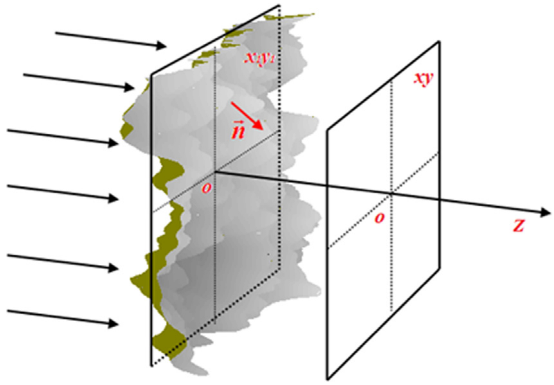


**Fig. 1.** Diagrammatic sketch for light scattering.

where, $A = \sqrt{1 + (\partial h/\partial x_1)^2 + (\partial h/\partial y_1)^2}$. Substituting the above formula into Eq. (1) we obtain an expression of wave field

$$U(x, y) = 1/4\pi \iint (1/Ar)[\exp(jk_0(-r - n_0 h))] \times [jk_0(-M/r + n_0 B) - M/r^2]\mathrm{d}x_1\mathrm{d}y_1. \quad (3)$$

where, $B = 1-(\partial h/\partial x_1)^2-(\partial h/\partial y_1)^2$, $M = (x-x_1)\partial h/\partial x_1+(y-y_1)\partial h/\partial y_1+B(h-z)$.

Then speckle phase $\phi$on observation plane can be acquired based on formula $\phi = arctg(u_i/u_r)$, where, $u_i$ and $u_r$ are the imaginary and real parts of the wave field, respectively.

### 2.2. Numerical calculations for speckle phase

First, we make random surface samples and acquire their height data. In order to study the evolutions of speckle phase with roughness, we grind three glass substrates of holographic plate with silicon carbide powders with sizes of 3.5 μm, 5 μm and 40 μm, respectively. All samples are measured with an atomic force microscope (AFM, PARK, Autoprobe CP, Contact mode, UL20 tip) and their AFM images with sizes 40 μm×40 μm and data points 256×256 are shown in Fig. 2. The images from left to right are labeled as sample No. 1, No. 2 and No. 3 and their height data will be used in following calculations. The roughness values of the three images are 0.0632 μm, 0.2304 μm and 0.6680 μm, respectively.

Based on Eq. (3) we calculate speckle fields of the three samples. In calculations, plane $(x_1, y_1, 0)$ is set at the location of average height of every sample. Refractive index $n_0$ of the random surfaces is 1.532. The sizes of observation plane are taken as 30 μm × 30 μm, smaller than that of the object plane, to avoid aperture diffraction effect. Incident light is assumed as He–Ne laser with wavelength 0.6328 μm. In order to obtain the evolutions of the speckle phase with scattering distances, we need to calculate the speckle phase at different distances. Statistical results show that the height maximum of all samples in Fig. 2 is 1.601 μm and then the minimum value of scattering distances is taken as 2 μm, extremely close to the random surfaces.

In order to understand the influences of aperture effect, we choose a glass substrate having the same size with the samples and calculate its phase distributions at scattering distance $z = 2$ μm. The result is shown in Fig. 3(a). Latticed image in Fig. 3(a) certifies the existence of aperture effect. Figs. 3(b) and (c) respectively show phase distributions at scattering distances $z = 2$ μm$+5.5\lambda$ and $z = 2$ μm$+10\lambda$. By comparing them, we can find that aperture effect becomes more evident with an increase of scattering distances. Besides, average gray value of Fig. 3(a) is almost same with that of Fig. 3(c), and absolutely different from that of Fig. 3(b). This phenomenon originates from phase space periodicity of plane light. In order to eliminate the influences of this phenomenon, scattering distances are all taken as $z = 2$ μm $+ N\lambda$, $N$ being integer in calculations.

In Figs. 4(a1), (b1) and (c1) we present all sample topography, just opposite to observation plane, and adopt different gray scales for clarity.

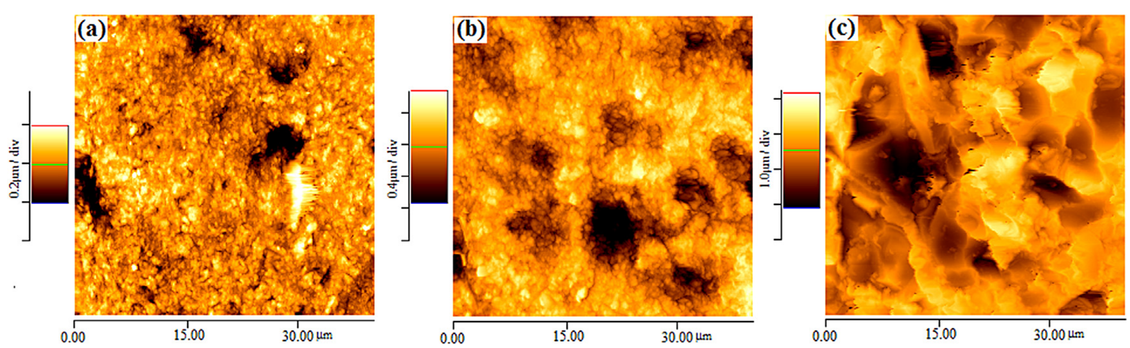


**Fig. 2.** AFM images for sample (a) No. 1, (b) No. 2 and (c) No. 3.

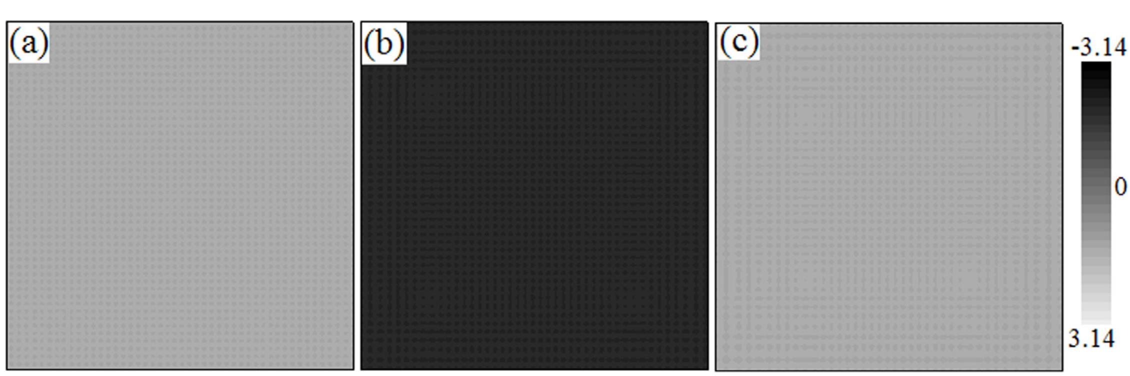


**Fig. 3.** Phase distributions of a glass substrate at scattering distances (a) $z = 2$ μm, (b) $z = 2$ μm $+ 5.5\lambda$ and (c) $z = 2$ μm $+ 10\lambda$.