Optics Communications 410 (2018) 836–840

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Cavity enhanced interference of orthogonal modes in a birefringent medium

Kiran Kolluru [a], Sudipta Saha [b], S. Dutta Gupta [a,*]

[a] School of Physics, University of Hyderabad, Hyderabad-500046, India
[b] Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur-741 246, India

ARTICLE INFO

Keywords:
Resonators
Birefringence
Interference
Polarization

ABSTRACT

Interference of orthogonal modes in a birefringent crystal mediated by a rotator is known to lead to interesting physical effects (Solli et al., 2003). In this paper we show that additional feedback offered by a Fabry–Perot cavity (containing the birefringent crystal and the rotator) can lead to a novel strong interaction regime. Usual signatures of the strong interaction regime like the normal mode splitting and avoided crossings, sensitive to the rotator orientation, are reported. A high finesse cavity is shown to offer an optical setup for measuring small angles. The results are based on direct calculations of the cavity transmissions along with an analysis of its dispersion relation.

1. Introduction

Interference of two or more resonances including their mixing has been one of the central themes of physics across its various branches [1–11]. The participating modes can belong to different subsystems as in atom-field, exciton–photon and opto-mechanical systems etc. [1,3, 6–8,10]. The modes can also be the independent modes of the same system. For example, they could be the clockwise and counter-clockwise modes of a fiber loop, spherical or disk resonators [4,5,12–14]. In metal/dielectric structures, these could be the surface plasmons at different interfaces, whispering gallery modes of metal–dielectric–metal micro and nano spheres [15,16]. One of the simple examples constitutes the mixing of orthogonal modes (ordinary and extraordinary waves) of a uniaxial birefringent crystal [17]. Obviously the cited examples constitute a very small subset of a very general and generic phenomenon of physics. Two different regimes of coupling, namely, weak and strong, have been identified and analyzed in detail [18]. The strong coupling regime is associated with an anticrossing or mode splitting of the resonant and near resonant modes [18–20]. The possibility of increasing the lifetime of the interacting modes in a dissipative system has been highlighted [19]. The mode splitting phenomenon has been observed in systems with large dissipation [21,22]. There have been many other interesting applications of the normal mode splitting phenomena. These include fast and slow light [8,17], detection of single-virus, single-molecules and nanoparticles [23,24], and also optical sensing [25]. Recently the splittings have been proposed as a tool for easier detection of the elusive transverse spin [26,27] in a gap plasmon guide [28].

In the context of the orthogonal modes in a birefringent crystal (BC), it was shown that a simple rotation of the detector horn can facilitate mixing of these modes [17]. The interference of these modes was shown to yield anomalous dispersion and superluminal light in an otherwise passive system. Note that usually anomalous dispersion is a characteristic of the absorption band of a material. The effect reported by Solli et al. [17] can further be magnified in a cavity. In this letter we study a Fabry–Perot (FP) cavity containing a BC (with optic axis perpendicular to the cavity axis) and a polarization rotator (R). We choose the working point at a wavelength where both the ordinary and the extraordinary waves are resonant. We show that the combination of BC with R in the cavity can lead to a regime of normal mode splitting and anti-crossing for specific rotator orientations. We present a vector formulation invoking the Jones matrices for the intra cavity elements to obtain the transmitted field and the dispersion relations. The numerical solutions of the transcendental dispersion equation are in full conformity with results for the intensity transmission. The fishnet in the $\theta - \lambda$ (rotator angle–wavelength) plane describing the peak transmitted intensities is explained by the closed-form solution of the dispersion equation. We also report an anti-crossing feature for specific points in the (θ-λ) plane. However, resolving the anti crossing features may be difficult because of the leakage-induced broadening of the split modes. Finally, we comment on the possible use of such an optical setup for

* Corresponding author.
E-mail address: sdgsp@uohyd.ernet.in (S.D. Gupta).

https://doi.org/10.1016/j.optcom.2017.11.056
Received 17 October 2017; Received in revised form 13 November 2017; Accepted 20 November 2017

measuring small angles, whereby the change in the rotator angle is mapped onto a well resolvable spectral separation in a high finesse cavity.

2. Mathematical formulation and results

Consider the setup shown in Fig. 1 comprising of a Fabry–Perot (FP) cavity with a intra-cavity rotator at $z = d_1$ and a birefringent crystal of length L. Let the cavity be illuminated by a plane polarized light propagating along the axis of the cavity (z-axis). The optic axis of the birefringent crystal is assumed to be oriented along y direction. We assume the incident field to be polarized at an angle α with respect to the x-axis. For identical lossless mirrors (with $\rho^2 + \tau^2 = 1$, where ρ and τ are the amplitude reflection and transmission coefficients of the mirrors, respectively) one can resort to the Jones matrix formalism to write the boundary conditions at $z = 0$ and $z = d$ (where $d = d_{free} + L$ is the total length of the cavity with $d_{free} = d_1 + d_2 + d_3$ giving the free space propagation inside) as follows.

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{z=0} = \tau \begin{pmatrix} E_x \\ E_y \end{pmatrix}_{in} + \rho \begin{pmatrix} E'_x \\ E'_y \end{pmatrix}_{z=0}, \quad (1)$$

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix}_{z=d} = \rho \begin{pmatrix} E_x \\ E_y \end{pmatrix}_{z=d}, \quad (2)$$

where the subscript 'in' represents the input fields and the unprimed (primed) field amplitudes refer to the forward (backward) propagating waves, respectively. Using Eqs. (1) and (2) one can write down the transmitted amplitudes (denoted by subscript tr) as follows,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{tr} = \tau^2 M_f (I - \rho^2 M_{total})^{-1} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_{in}, \quad (3)$$

where I is the 2×2 identity matrix and $M_{total} = M_b\, M_f$ gives the round-trip Jones matrix for the cavity with M_f (M_b) representing the Jones matrix for forward (backward) pass as follows [11,29,30]

$$M_b = M_{free} R(\theta) M_{BC}, \quad (4)$$

$$M_f = M_{BC} R(\theta) M_{free}, \quad (5)$$

with

$$M_{free} = e^{ik_0 d_{free}}\, I, \quad M_{BC} = Diag[e^{ik_0 n_o L}, e^{ik_0 n_e L}]. \quad (6)$$

In Eqs. (4)–(6) k_0 is the free space propagation constant, $R(\theta)$ is the standard rotation matrix and n_e (n_o) is the refractive index of the BC for the extraordinary (ordinary) wave. Introducing notations κ, $\tilde{\delta}$ as

$$\kappa = \frac{n_e + n_o}{n_e - n_o}, \quad (7)$$

$$\tilde{\delta} = k_0 (n_e - n_o) L, \quad (8)$$

the final form of M_{total} can be written as

$$M_{total} = e^{2ik_0 d_{free}} e^{i\kappa\tilde{\delta}} \begin{pmatrix} \cos^2\theta e^{-i\tilde{\delta}} - \sin^2\theta e^{i\tilde{\delta}} & -\sin 2\theta \cos\tilde{\delta} \\ \sin 2\theta \cos\tilde{\delta} & \cos^2\theta e^{i\tilde{\delta}} - \sin^2\theta e^{-i\tilde{\delta}} \end{pmatrix}. \quad (9)$$

It is clear that without the rotator ($\theta = 0°$), the x and y components of the field experience phase shifts,

$$2k_0(d_{free} + n_o L) = 2n\pi, \quad 2k_0(d_{free} + n_e L) = 2m\pi, \quad (10)$$

respectively, defining the ordinary and the extraordinary modes. In Eq. (10) m and n are integers. As can be seen from Eq.(10) coincidence of the resonances for both the modes is governed by L, since both have same contribution from free space propagation. We pick the cavity parameters such that both the ordinary and the extraordinary waves are resonant at the same wavelength, say at $\tilde{\lambda}_0$. To be specific, for a given length L we choose $\tilde{\lambda}_0$ such that $\tilde{k}_0(n_e - n_o)L = (m-n)\pi = q\pi$, $\tilde{k}_0 = 2\pi/\tilde{\lambda}_0$. For example, for $L = 10$ μm, $n_e = 2.28$, $n_o = 2.25$ and $q = 1$, we get $\tilde{\lambda}_0 = 0.6$ μm.

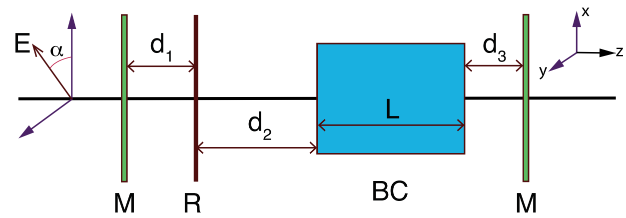

Fig. 1. (Color online) Schematics of the FP cavity with the mirrors M, the intra-cavity rotator R and the birefringent crystal BC with optic axis along the y-axis. Parameters are as follows: $\rho = 0.95$ (for both the mirrors), $L = 10$ μm, $d_{free} = d_1 + d_2 + d_3 = 15$ μm, $n_e = 2.28$, $n_o = 2.25$.

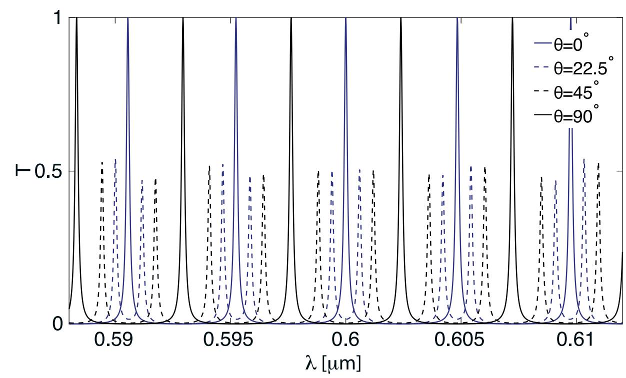

Fig. 2. (Color online) Intensity transmission T as a function of λ for varying rotator angles namely, $\theta = 0°$ (solid blue line), 22.5° (dashed blue line), 45° (dashed black line), 90° (solid black line). Parameters are as follows, $\rho = 0.95$, $L = 10$ μm, $d_{free} = 15$ μm, $n_e = 2.28$, $n_o = 2.25$, $\alpha = 0°$.

In what follows we present the numerical results for the intensity transmission coefficient T ($= T_x + T_y = |E_x|^2 + |E_y|^2$ at the exit face) for an incident field with unit magnitude. We have chosen the following parameters for the numerical calculations: $\rho = 0.95$, $L = 10$ μm, $d_{free} = 15$ μm, $n_o = 2.25$ and $n_e = 2.28$. For the chosen set, the two orthogonal modes are degenerate at $\tilde{\lambda}_0 = 0.6$ μm (see Fig. 3(a) below). For an incident field polarized along the x-axis ($\alpha = 0$), T as a function of λ for different orientations of the rotator is shown in Fig. 2. We have presented results for a range of wavelengths and for four different values of θ, namely, $\theta = 0°, 22.5°, 45°$ and $90°$. Note that the curves for $\theta = 90°$ represent the orthogonal extraordinary modes. Further it can be seen that the mode splitting is asymmetric (with sidebands with unequal heights) for modes that are near degenerate. A deeper insight can be obtained by analyzing the corresponding dispersion relation which is presented below.

In contrast, in absence of the rotator inside the cavity ($\theta = 0°$), even for tilted incident field (for finite α) the splittings presented in Fig. 2 are missing. For reference we have shown the ordinary (extraordinary) modes for x- (y-) polarized incident wave in absence of the rotator ($\theta = 0°$) in Fig. 3(a). The case for $\alpha = 45°$ is shown in Fig. 3(b), where the intensity transmission results from a superposition of the x– and y–components of the field. Clearly the peaks correspond to the resonances of the ordinary and extraordinary modes, which are not degenerate near $\lambda = 0.6147$ μm. Thus the origin of splitting can be traced to multiple passes through the rotator (at angles other than 0 or multiple of $\pi/2$ with respect to the optic axis) inside the cavity.

We now show how a high finesse cavity can facilitate the measurement of small angles by optical means. Usually the finesse $\mathcal{F}$ of a Fabry–Perot cavity is given as $\mathcal{F} = \frac{\pi\sqrt{\rho}}{1-\rho}$. It is related to the quality factor of the cavity. The higher the value of $\mathcal{F}$, the greater is the quality factor. To this end we have plotted the transmission spectra for a small rotator angle, say $\theta = 0.5°$ for three different values of the finesse (see Fig. 4). While, width of the split resonances mask the splittings for a low finesse