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Linear stability analysis of coupled parallel flexible plates in an axial flow

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Abstract

We study here the linear stability of N identical flexible plates with clamped–free boundary conditions forced by a uniform parallel flow. Flow viscosity and elastic damping are neglected, and the flow around the plates is assumed potential. The shedding of vorticity from the plates' trailing edges is accounted for by introducing a force-free wake behind each plate. A Galerkin method is used to compute the eigenmodes of the system. We are interested in the effects of the number of plates and their relative distance on the stability property of the state of rest, as well as in the nature and structure of the coupled states. Detailed results are presented for the cases $N = 2$, $N = 3$ and $N \gg 1$.

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1. Introduction

The flapping motion of flexible plates or flags placed in a parallel imposed flow has been the subject of a large number of both experimental (Zhang et al., 2000; Shelley et al., 2005) and numerical studies (Connell and Yue, 2007; Zhu and Peskin, 2002). The flapping motion results from the fluttering instability of the plate's straight position under the competing effects of solid inertia, aerodynamic pressure forcing and solid flexural rigidity. In that regard, the nature of the instability is similar to that of flow-conveying pipes (Païdoussis, 1998). Beyond the fundamental interest of this instability and the resulting flapping motions, this mechanism is at the origin of the flapping of flags in the wind and is also of interest for engineering (Watanabe et al., 2002) and biomedical applications (Huang, 1995; Balint and Lucey, 2005). Hydroelastic instabilities of parallel-plate assemblies are also important in the study of industrial cooling systems, as found in some nuclear reactors (Miller, 1960; Kim and Davis, 1995; Guo and Païdoussis, 2000a).

Several models have been proposed to understand the instability threshold in the case of a single elastic plate. Shelley et al. (2005) considered the one-dimensional linear stability of a flag of infinite span and length under parallel flow

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forcing. Lemaitre et al. (2005) used a slender-body approximation to study the linear stability of long elastic ribbons. Kornecki et al. (1976) studied the linear stability of a finite-length cantilevered plate of infinite span, using a representation of the vortical wake as a vortex sheet (Theodorsen, 1935; Bisplinghoff et al., 1955), and Argentina and Mahadevan (2005) used a similar approach to study the influence of in-plane tension and finite span on the stability threshold. Guo and Païdoussis (2000b) considered the stability of a finite-length two-dimensional plate in a channel with various boundary conditions, using a simpler representation of the wake. Eloy et al. (2007) used the same approach to investigate the influence of three-dimensional effects on the stability of a finite-length cantilevered plate.

In the case of multiple parallel elastic plates, the imposed flow acts both as a forcing and a coupling mechanism between the motion of the plates. When the distance between the plates is of the order of or less than their length, it is expected that their dynamics will be strongly coupled. Zhang et al. (2000) generalized their soap-film experiment to two parallel filaments and showed that the filaments' dynamics became coupled and that, depending on their relative distance, in-phase and out-of-phase modes could be observed (Fig. 1). This observation was confirmed by Jia et al. (2007) who also considered filaments of various lengths. Depending on the length of the filaments, they observed either a transition from out-of-phase to in-phase flapping when bringing the filaments close together, or a persistence of the out-of-phase mode even at small separation. These results suggest that the nature of the observed regime is strongly dependent on more than just the relative distance between the different plates or filaments. Numerical simulations (Zhu and Peskin, 2003; Farnell et al., 2004; Huang et al., 2007; Michelin and Llewellyn Smith, 2009) have also reported the existence of these two classes of regimes. Tang and Païdoussis (2009) presented a numerical study of the stability criteria for the in-phase and out-of-phase modes. More recently, Schouveiler and Eloy (2009, private communication) considered three and more coupled plates in a wind-tunnel experiment and observed various mode structures (Fig. 1). In particular, in the symmetric mode, the middle plate was observed to remain still. Guo and Païdoussis (2000a) studied the linear stability of an infinite number of parallel plates of finite aspect ratio, clamped on their side edges and free at the leading and trailing edges. They however focused only on one particular type of modes (the out-of-phase mode, where the motions of two consecutive plates are always opposite), using the resulting symmetries to significantly reduce the complexity of the system.

The purpose of the present study is to propose a linear stability analysis of N coupled flexible plates of infinite span, clamped at their leading edge in a uniform imposed flow. We investigate the existence and nature of coupled modes, that were previously observed experimentally and numerically (Zhang et al., 2000; Zhu and Peskin, 2003; Jia et al., 2007), as well as the influence of the plates' separation on the stability of the trivial state of rest and the structure of the linearly dominant flapping modes. In Section 2, the double-wake method used in Guo and Païdoussis (2000b) and Eloy et al. (2007) for a single plate is extended to the case of an arbitrary number of equidistant parallel plates. In Sections 3 and 4, results on the linear stability of respectively two and three plates and the structure of the most unstable modes are discussed. Section 5 proposes an overview of expected results at larger N as well as a discussion of the influence of N on

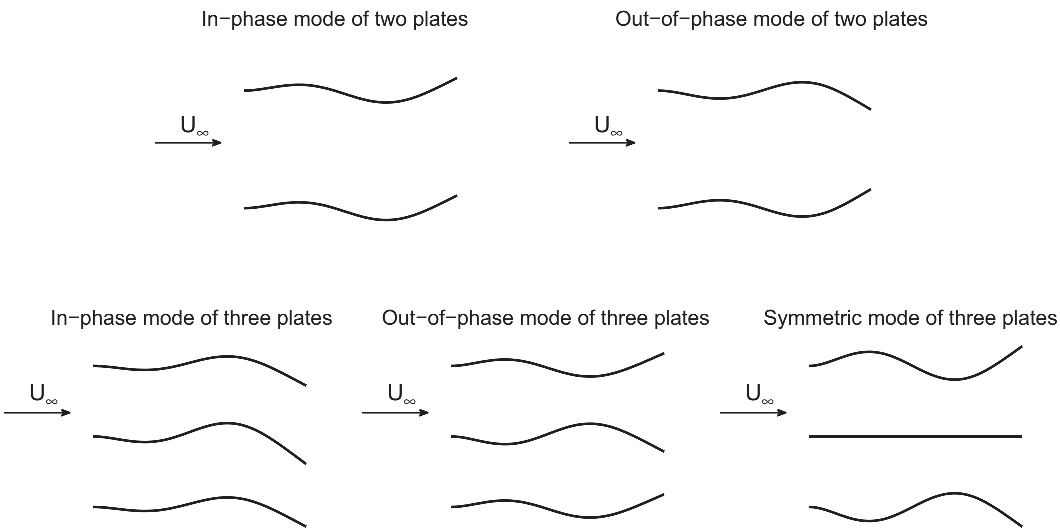

Fig. 1. (Top) Sketch of the in-phase and out-of-phase modes observed experimentally for two plates (Zhang et al., 2000; Jia et al., 2007). (Bottom) Sketch of the in-phase, out-of-phase and symmetric modes observed experimentally for three plates (Schouveiler and Eloy, 2009, private communication).