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Periodically modulated single-photon transport in one-dimensional waveguide

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ABSTRACT

Single-photon transport along a one-dimension waveguide interacting with a quantum system (e.g., two-level atom) is a very useful and meaningful simplified model of the waveguide-based optical quantum devices. Thus, how to modulate the transport of the photons in the waveguide structures by adjusting certain external parameters should be particularly important. In this paper, we discuss how such a modulation could be implemented by periodically driving the energy splitting of the interacting atom and the atom–photon coupling strength. By generalizing the well developed time-independent full quantum mechanical theory in real space to the time-dependent one, we show that various sideband-transmission phenomena could be observed. This means that, with these modulations the photon has certain probabilities to transmit through the scattering atom in the other energy sidebands. Inversely, by controlling the sideband transmission the periodic modulations of the single photon waveguide devices could be designed for the future optical quantum information processing applications.

1. Introduction

Single-photon propagation is one of the basic and important subjects in quantum optics. It is related to the designs and fabrications of various optical quantum devices for optical quantum information processings [1–3]. Recently, many theoretical and experimental works have been demonstrated to investigate the single-photon transport along a one-dimension waveguide with aside one- and multi- atoms as the scatters [4–9]. These investigations are directly related to various single-photon quantum device applications to implement, e.g., the single-photon routers, switches, and detectors, etc., [10–16], as well as quantum communications and quantum information applications [17–21]. Note that, almost all these works are based on a time-independent quantum theory, i.e., the Hamiltonians of the considered systems are time independent, and thus can only describe the elastic scatterings of the photons in the waveguide by the aside atom(s).

However, manipulatable single-photon devices are usually necessary for many practical applications, such as the quantum Zeno switches [22]. Therefore, the investigation of how to modulate the transport of the photons along the waveguide-atom structures by controlling certain external parameters should be meaningful. Physically, these modulations can be applied to either the energy splitting(s) of the scattering atom(s) or the photon–atom interaction, or both of them. For example, in a recent experiment [23] the famous dynamical Casimir effect was verified by probing the sideband photons, generated by the microwave propagating along a coplanar waveguide terminated by a superconducting quantum interference device with fast changing magnetic flux.

It is noted that the time-dependent transport problem is usually encountered for the electron transport along the electronic waveguide in mesoscopic physics, and the relevant theory [24–28], including the so-called Floquet theory for periodic modulation [26–30], has been developed well by directly solving the time-dependent Schrödinger equation. A typical deduction for this theory is, due to the inelastic scatterings the electrons could be transmitted/reflected into the various energy sidebands (with the zero-sideband describing the elastic scattering of the electrons). As a consequence, the electronic transport could be modulated, in principle, from one sideband to the others.

Similar to the time-dependent electronic waveguide transport theory, in this paper we will develop a time-dependent single-photon transport theory to describe the photons propagating in the optical waveguides with certain time-modulations. Certainly, due to the present inelastic scatterings, the photons can also be propagated in various

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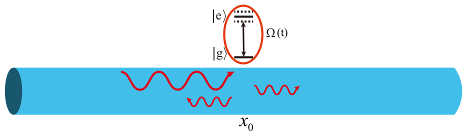

Fig. 1. A single photon transporting along a one-dimension waveguide is scattered by a two-level atom (at x_0) with the energy splitting between $|g\rangle$ and $|e\rangle$ being periodically modulated.

energy sidebands, and thus the total transmitted/reflected probability of the photon should be the sum of the ones in all the possible sidebands. Physically, the desired modulations could be achieved by adjusting the atomic level-splittings or the atom–photon coupling strength. As a consequence, the transport of the photons along the designed waveguides can be controlled at a single-photon level.

The paper is organized as follows. In Section 2, we present our model by considering a single waveguide photon scattered by a two-level atom with the periodic modulating energy splitting. With such a modulation we show that the photons could transmit through the atom in certain energy sidebands. In Section 3, keeping the transition frequency of the atom unchanged, we investigate how to control the transport of the photon by using the periodically modulated photon–atom interaction. In this case, we find that the transmission of the photon is mainly along the $n \neq 0$ sideband (due to the inelastic scattering), and the transmission in the zero-sideband (related to the elastic scattering) is negligible. Finally, in Section 4 we summarize our work and discuss the potential applications of the time-dependent single photon transport theory developed here.

2. A single waveguide photon scattered by a two-level atom with periodic modulated transition frequency

At the first, let us consider a simplest model, i.e., a single-photon with the fixed frequency transporting along a one-dimension waveguide and being scattered by an ideal two-level atom (i.e., without any atomic decay), whose eigenfrequency is periodically modulated. The system is sketched in Fig. 1, wherein the energy splitting between the ground state $|g\rangle$ and the excited state $|e\rangle$ of the atom, locating at x_0, is periodically modulated. The Hamiltonian of the system can be written as ($\hbar = 1$) :

$$H = \int dx \left[c_R^\dagger(x)(-iv_g \frac{\partial}{\partial x})c_R(x) + c_L^\dagger(x)(iv_g \frac{\partial}{\partial x})c_L(x)\right] + \int dx V\delta(x-x_0)\left[c_R(x)\sigma^+ + c_L(x)\sigma^+ + H.c.\right] + \Omega(t)\sigma^+\sigma^-. \tag{1}$$

Here, $c_R^\dagger(x)$ ($c_R(x)$) and $c_L^\dagger(x)$ ($c_L(x)$) are the bosonic creation (annihilation) operators of the single-photon propagating right and left directions, respectively. v_g is the group velocity of the photon, V is the coupling strength between the waveguide photon and the atom, and $\sigma^+(\sigma^-)$ the atomic raising (lowering) ladder operator. The atomic transition frequency Ω between the ground and excited states is now periodically modulated, i.e., $\Omega(t) = \Omega[1 + f\cos(\omega t)]$ with $f \ll 1$ being the modulated amplitude and ω the modulated frequency. The atom–photon coupling strength V is kept unchanged and the dissipations of the system are neglected also for simplicity.

The generic solution to the time-dependent Schrödinger equation with the Hamiltonian (1) can be expressed as

$$|\Psi\rangle = \int dx \left[\phi_R(x,t)c_R^\dagger(x) + \phi_L(x,t)c_L^\dagger(x)\right]|\varnothing\rangle + e(t)\sigma^+|\varnothing\rangle, \tag{2}$$

with $|\varnothing\rangle$ being the vacuum state, i.e., without any photon in the waveguide and the atom stays at its ground state $|g\rangle$, and $\phi_{R/L}(x,t)$ and $e(t)$ standing for the time-dependent probabilistic amplitudes of the photon propagating along the R/L direction and the atomic excitation, respectively. The time-dependent coefficients in the above wave function are determined by the following equations:

$$i\frac{\partial}{\partial t}\phi_R(x,t) = -iv_g\frac{\partial}{\partial x}\phi_R(x,t) + V\delta(x)e(t), \tag{3}$$

$$i\frac{\partial}{\partial t}\phi_L(x,t) = iv_g\frac{\partial}{\partial x}\phi_L(x,t) + V\delta(x)e(t) \tag{4}$$

$$i\frac{\partial}{\partial t}e(t) = \Omega[1 + f\cos(\omega t)]e(t) + V\left[\phi_R(0,t) + \phi_L(0,t)\right]. \tag{5}$$

As the incident single-photon is now scattered by a time-dependent atom and thus its energy should be no longer conservation. This implies that the photon could be transmitted/reflected into the different energy states, i.e., energy sidebands. The above probabilistic amplitudes of the photon propagating along the R/L direction could be taken generically as

$$\phi_R(x,t) = \theta(-x+x_0)e^{i(q_0x-\omega_0t)} + \theta(x-x_0)\psi_R(x,t), \tag{6}$$

$$\phi_L(x,t) = \theta(-x+x_0)\psi_L(x,t), \tag{7}$$

where $\psi_R(x,t)$ and $\psi_L(x,t)$ stand for the transmitted and reflected parts of the scattered photon, respectively. Also, ω_0 is the frequency of the incident photon with the wave vector $q_0 = \omega_0/v_g$.

Without loss of the generality, we take $x_0 = 0$ for simplicity. By substituting Eqs. (6) and (7) into Eqs. (3) and (4), we have

$$\psi_R(0,t) = \psi_L(0,t) + e^{-i\omega_0 t}, \tag{8}$$

$$Ve(t) = iv_g\psi_L(0,t). \tag{9}$$

Furthermore, with Eq. (5) we get

$$\frac{\partial}{\partial t}e(t) = -i\Omega[1 + f\cos(\omega t)]e(t) - \frac{V^2}{v_g}e(t) - iVe^{-i\omega_0 t}. \tag{10}$$

A particular solution to the homogeneous differential equation on $e(t)$ reads

$$e(t) = e^{-i\Omega t - \frac{V^2}{v_g}t}e^{-i\frac{f\Omega}{\omega}\sin(\omega t)}. \tag{11}$$

By using the Jacobi–Anger expansion [31]

$$e^{iu\sin x} = \sum_n J_n(u)e^{inx}, \tag{12}$$

with $J_n(u)$ being the first kind Bessel function of the n-order, the generic solution to the Eq. (10) reads

$$e(t) = \sum_{n,l}\frac{VJ_l(\frac{f\Omega}{\omega})J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma}e^{-i\omega_n t}, \tag{13}$$

with $\Delta = \omega_0 - \Omega$, $\gamma = V^2/v_g$ and $\omega_n = \omega_0 + n\omega$. Here, Δ and γ are the detuning and the effective coupling strength between the photon and the periodically-modulated atom, respectively. As a consequence,

$$\psi_L(x,t) = \sum_n e^{-i(q_nx+\omega_nt)}\left[\sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega})J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma}\right], \tag{14}$$

$$\psi_R(x,t) = \sum_n e^{i(q_nx-\omega_nt)}\left[\sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega})J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} + \delta_{n,0}\right], \tag{15}$$

with $q_n = \omega_n/v_g$. It is seen that many energy sidebands appear in the reflected and transmitted coefficients of the scattered photon. With Eqs. (6) and (7) one can easily see that the quantities defined in the square brackets in Eqs. (14) and (15) are just the reflected and transmitted amplitudes in the nth sideband, i.e.,

$$r_n = \sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega})J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma}, \tag{16}$$

$$t_n = \sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega})J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} + \delta_{n,0}. \tag{17}$$