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Effect of spanwise flexibility on flapping wing propulsion

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Abstract

A water tunnel study of the effect of spanwise flexibility on the thrust, lift and propulsive efficiency of a rectangular wing oscillating in pure heave has been performed. The thrust and lift forces were measured with a force balance, and the flow field was measured with a Particle Image Velocimetry system. Introducing a degree of spanwise flexibility was found to be beneficial. For Strouhal numbers greater than 0.2, a degree of spanwise flexibility was found to yield a small increase in thrust coefficient, and a small decrease in power-input requirement, resulting in higher efficiency. In this case, a moderately stronger trailing-edge vortex system was observed. Introducing a far greater degree of spanwise flexibility, however, was found to be detrimental. A large phase delay of the wing tip displacement was observed, leading to the root and tip moving in opposite directions for a significant portion of the flapping stroke. Vorticity of opposing sign was observed to be shed from the root and tip, resulting in a weak and fragmented vorticity pattern. The thrust coefficient was observed to be significantly reduced, and the efficiency diminished. It is noted that the range of Strouhal numbers for which spanwise flexibility was found to offer benefits overlaps the range found in nature, of $0.2 < Sr < 0.4$. From a design aspect, flexibility may benefit flapping-wing Micro Air Vehicles both aerodynamically and in the inherent lightness of flexible structures.

Keywords: Vortex flows; Flapping wings; Heave; Thrust; Propulsive efficiency; Wing flexibility.

1. Introduction

There is great interest in small (wing-span less than six inches) radio-controlled aircraft known as *Micro Air Vehicles* (MAVs). Many applications have been suggested for a MAV carrying a miniature video camera or other sensing device. One specific example from industry would be the surveying of a chemical refinery following a storm. Civil and military applications have also been suggested. The majority of MAV applications require great manoeuvrability, and some demand the ability to hover. The wonderful agility of birds, bats, and insects has led to the design of flapping wing MAVs (Spedding and Lissaman, 1998; Shyy et al., 1999). For this reason, research on flapping wing propulsion has attracted considerable attention recently. Because of the small length scales involved, the Reynolds number is low, typically of order 10^3–10^5.

The origin of thrust for an oscillating airfoil was found by Knoller (1909), and later and independently by Betz (1912). The *Knoller–Betz* effect was demonstrated in a wind tunnel experiment by Katzmayr (1922). Following this, a number of

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Nomenclature

a	amplitude
b	wing span
c	chord length
C_T	thrust coefficient
C_P	power coefficient
F	force per unit span
h	root amplitude (a_{ROOT}/c)
k_G	Garrick reduced frequency ($\pi fc/U_0$)
Re	Reynolds number ($U_0 c/v$)
s	displacement
Sr	Strouhal number ($2fa_{\mathrm{MID}}/U_0$)
t	time
T	thrust per unit span; period
U_0	freestream velocity
v	wing root velocity
η	propulsive efficiency
μ	viscosity
ρ	density
φ	tip phase angle
ω	heave angular frequency
Ω	vorticity

Subscripts

x	streamwise direction
y	lateral direction
z	spanwise direction

theoretical and numerical models of oscillating airfoils were developed. These early models, by Garrick (1936), Lighthill (1970), and Wu (1971), were of thin, 2-D airfoils, oscillating in inviscid flow. Consequently, they significantly overestimated the propulsive efficiency (Isogai et al., 1999; Ramamurti and Sandberg, 2001) in the low-Reynolds-number separated flows observed in natural flight, and in the flight of MAVs (Jones et al., 2001a). More recently, Navier–Stokes simulations (Tuncer and Platzer, 1996) have yielded more accurate flow pattern and propulsive efficiency predictions.

Experiments have focused on rigid airfoils (Koochesfahani, 1989; Jones et al., 1998; Lai and Platzer, 1999), where the effects of oscillation mode (e.g. pure heave, pure pitch, Emblemsvag et al., 2003; coupled heave and pitch, Anderson et al., 1998), (Hover et al., 2004) (e.g. sine wave, triangle wave), and aspect ratio (Jones et al., 2002) have been investigated. The special case of hovering flight has also received attention (Freymuth, 1990; Sunada et al., 2001). The effect of wing stiffness, in either the chordwise or spanwise direction, is relatively unexplored. This is surprising given the importance of flexibility to fish (Triantafyllou et al., 2000), and the finding of intricate variations in the stiffness of insect wings (Wootton, 1981)—though the role of flexibility in insect flight is still unclear (Maxworthy, 1981).

The effect of chordwise flexibility for an airfoil in heave at low Reynolds numbers has been studied by Heathcote and Gursul (2006). A schematic of the 2-D experiment is shown in Fig. 1(a). The airfoil comprises a tear-drop solid aluminium leading edge followed by a flexible steel plate. The airfoil is rigid in the spanwise direction. The thin plate deforms under fluid dynamic forces, making an angle θ with the freestream direction. The thrust force on the wing has been measured over a range of plate stiffnesses and oscillation frequencies. The variation of thrust coefficient with stiffness is shown in Fig. 1(b). Each of the three series corresponds to a different frequency. For each frequency, intermediate plate stiffness yields the greatest thrust force. Particle Image Velocimetry measurements revealed a correspondingly stronger jet vortex pattern. Chordwise flexibility is also found to bear efficiency benefits (Heathcote et al., 2004).

The progression to a study of spanwise flexibility follows naturally. Spanwise flexibility is of interest because the wings of birds and the fins of fish and aquatic mammals are flexible. One question is whether spanwise flexibility is beneficial to bird flight, or whether it is a limitation, due to the finite stiffness of the bone structure of the wing. Liu and Bose (1997) studied the effect of spanwise flexibility on the flukes of an immature fin whale, using inviscid calculations. The phase of the flexing motion relative to the heave was found to be a key parameter in determining the thrust and efficiency characteristics of the fin. In-phase motions yielded a benefit in efficiency and a significant increase in thrust. Out of phase motions were found to be detrimental. The subject of flexibility is particularly relevant to the design of miniature flapping wing aircraft, for which weight is a key restraint: light wings are inherently flexible.

The purpose of the present experimental study is to measure the effect of spanwise flexibility on the thrust and efficiency characteristics of a rectangular wing oscillated in heave. The heave amplitude, h, where $h = a_{\mathrm{ROOT}}/c = 0.175$, is constant for all experiments. Three additional dimensionless parameters may be defined: the Reynolds number, Garrick frequency, and Strouhal number based on the amplitude of the mid-span ($z = b/2$):

$$\mathrm{Re} = \frac{\rho U_0 c}{\mu}, \quad k_G = \frac{\pi fc}{U_0}, \quad \mathrm{Sr} = \frac{2fa_{\mathrm{MID}}}{U_0}.$$