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Analytical properties of the effective refractive index

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ABSTRACT

The propagation of a plane wave through a periodic layered system is considered in terms of the effective parameters. The problem of introduction of effective parameters is discussed. It was demonstrated that although the effective admittance cannot be introduced, it is possible to introduce the effective refractive index, which tends toward the Rytov value when the system size increases. It was shown that the effective wave vector derivative is an analytical function of frequency. In particular, the Kramers–Kronig-like relations for real and imaginary parts of the effective wave vector derivative were obtained. The Kramers–Kronig-like relations for the effective refractive index were also considered. The results obtained numerically were proved by exact solution of Maxwell's equations in the specific case of an "equi-impedance" system.

1. Introduction

In the long wave approximation, it is convenient to replace an inhomogeneous composite material with a homogeneous medium, whose electromagnetic properties result from an averaging of the local electromagnetic field. This approach is usually called a *homogenization procedure*.

However, in some particular cases, the introduction of conventional effective parameters—electric permittivity ε_{eff} and magnetic permeability μ_{eff} may look physically incorrect. For example, the use of Rytov's effective parameters ε_{eff} and μ_{eff} [1–7] for a periodic layered sample lead to contradictions [8–12]. Let us consider a finite periodic sample (whose unit cell consists of two layers) composed of an even number of layers (see Fig. 1). In the sample, the first and the last layers are made of different materials. Thus, the sample is not symmetric with respect to the reversal of light incidence direction [8]; even in the lossless case, the phases of reflection coefficients for waves incident from the right (r_R) and from the left (r_L) are different.[1]

However, for a homogeneous layer, the phases should be the same. Calculations directly show that the difference in r_R and r_L is of the first order of d/λ, where λ is wavelength and d is the thickness of a single layer. Therefore, the homogenization procedure for a layered system in terms of ε_{eff} and μ_{eff} is correct only in the static case ($\lambda = \infty$) [8].

For a sample composed of an odd number of layers, the effective parameters' dependency on sample size was demonstrated in [8,9]. Fig. 2 displays the differences between parameters retrieved for a finite sample composed of an odd number of layers and the Rytov parameters. The peaks appearing at certain points do not vanish with the increase in system size. Thus, the retrieved parameters describe the sample instead of some homogeneous material.

Despite the fact that ε_{eff} and μ_{eff} cannot be correctly introduced beyond the static approximation, it is possible to introduce an effective refractive index. The calculations in the long wave approximation show that an effective wave vector (retrieved from scattering parameters) tends toward a constant predicted by Rytov as the number of layers increases [8–11]. Fig. 3 from [9] illustrates the convergence of the effective refractive index to the value predicted for infinite size.

The problem of introducing effective parameters beyond the static approximation is reduced to two independent problems: definition of the effective refractive index and description of the boundary condition. Previously, the regularization of effective impedance with the help of additional surface currents [13] or an additional effective layer on the border [14] was considered an approach for fixing the boundary condition problem. However, this problem is still far from being solved.

Now we will move away from the problem of boundary

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[1] In the lossy case these coefficients are also different in amplitude.

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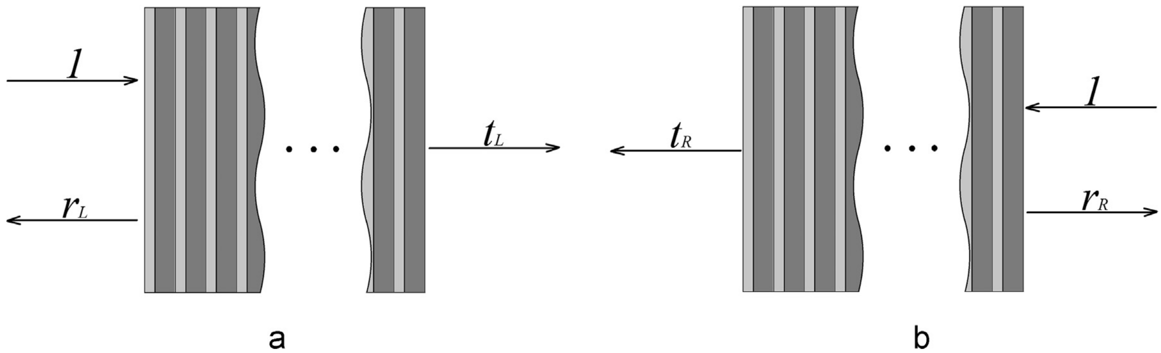

Fig. 1. A finite periodic sample composed of an even number of layers. The unit cell consists of two layers. Light incidence from the left (a) and from the right (b) to the sample is shown.

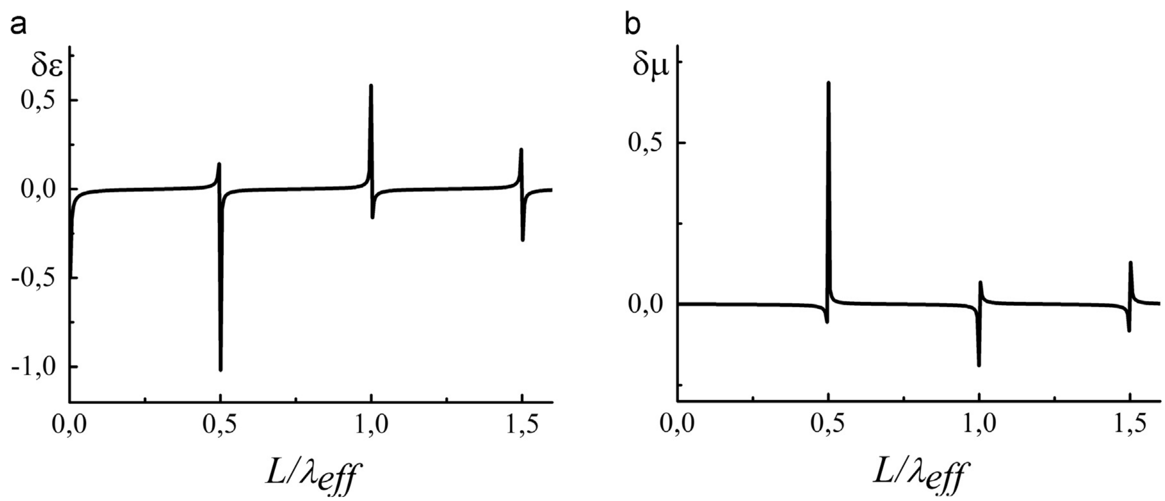

Fig. 2. Sample thickness dependences for (a) $\varepsilon_{eff} - \varepsilon_{eff}^{Ryt}$ and (b) $\mu_{eff} - \mu_{eff}^{Ryt}$ for a periodic system with an odd number. The sample thickness is in effective wavelength $\lambda_{eff} = \lim_{L\to\infty}\left(2\pi/k_{eff}\right)$ units. Parameter values: $\varepsilon_1 = 2$, $\varepsilon_2 = 3$.

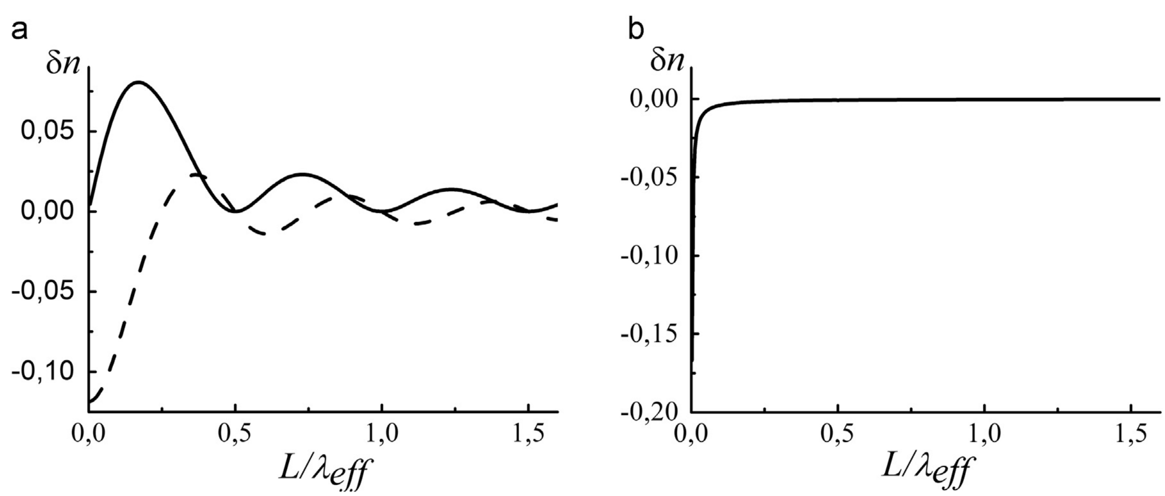

Fig. 3. Sample thickness dependences for $\delta n_{eff} = n_{eff} - n_{eff}^{Ryt}$ for a periodic system with an even (a) and an odd (b) number of layers. The sample thickness is in effective wavelength $\lambda_{eff} = \lim_{L\to\infty}\left(2\pi/k_{eff}\right)$ units. Parameter values: $\varepsilon_1 = 2$, $\varepsilon_2 = 3$. Real and imaginary parts are shown by solid and dashed curves, respectively.

conditions and discuss only the properties of the effective refractive index n_{eff} or effective wave vector k_{eff}, which is

$$k_{eff} = n_{eff}\frac{\omega}{c},$$

where ω is the wave frequency and c is the speed of light. In the present communication we are discussing the possibility of introducing an effective refractive index for periodic systems and the analytical properties of the effective wave vector.

It should be noted that the effective wave vector is important by itself because it describes eigenmode properties. In particular, the refraction law (generalization of Snell's law) is described by the refractive index only. In addition, a number of homogenization procedures have been proposed [15–17], which use the effective refractive index as the only effective parameter needed.