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Dynamic evolution of coherent vortex dipole in atmospheric turbulence

Jinhong Li [a,*,1], Jun Zeng [a,b,1]

[a] Department of Physics, Taiyuan University of Science and Technology, Taiyuan 030024, China
[b] School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China

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ABSTRACT

The analytical expression for the cross-spectral density function of Gaussian Schell-model (GSM) beams with coherent vortex dipole (CVD) propagating through atmospheric turbulence is derived, which enables us to study the evolution process of CVD propagating through atmospheric turbulence, where the influences of the beams parameters and atmospheric turbulence parameters on the ratio of critical off-axis distance to the waist width are stressed. It shows that the evolution process of the CVD depends on the off-axis distance. The larger the off-axis distance is, the more the number of CVD is. When the off-axis distance is zero, the position of coherent vortices with positive and negative topological charge of CVD propagating through atmospheric turbulence is always symmetry. When the off-axis distance is big enough, compared with the situation at source plane, the orientation of the positive coherent vortex of inherent CVD and negative coherent vortex of that rotates 180° in the far field. The larger the structure constant and the waist width are, as well as the smaller the spatial correlation length and the inner scale are, the smaller the ratio a_c/w_0 is. Besides, the ratio a_c/w_0 will no longer change when the spatial correlation length or the inner scale increases to a certain value, whereas the outer scale has no effect on the ratio.

1. Introduction

Much interest has been exhibited in the focusing of vortex beams carrying orbital angular momentum and wavefront dislocations due to their theoretical importance and potential applications in micronsized particle manipulation, photon counting, optical data storage and quantum communications, etc. [1–9]. In recent years, the research object of vortex beams has been from single vortex beams [10–14] extended to double vortex beams with opposite topological charges (vortex dipole). Furthermore, the propagation properties of novel vortex beams in free space were dealt with numerically in [15]. Compared with single vortex beam, coherent vortex dipole (CVD) has more abundant dynamic characteristics and potential application values [16–20]. As pointed out by Indebetouw, a pair of coherent vortices with opposite topological charges of the CVD propagation through free space attracts each other and this pair can collide and annihilate, in addition, the two vortices of equal topological charges in the far field have rotated 90° from their original angular position [16]. Freund have suggested that the CVD with opposite topological charges will reappear in the far field after the annihilation [17]. Gao et al. studied the evolution of the CVD with opposite topological charges diffracted by a half screen and found that compared with the case of the CVD evolution in free space, multi-pairs of the CVD may take place in the diffracted field, and the annihilation fashion depends on the off-axis distance [18]. Roux and Yan subsequently reported the transmission of the CVD in a graded index medium and an astigmatic lens [19,20]. However, the work mentioned above did not refer to the dynamic evolution of the CVD in atmospheric turbulence. In this paper, taking the Gaussian Schell-model (GSM) beams with the CVD as an example of partially coherent vortex beams, we have studied the dynamic evolution of CVD in atmospheric turbulence, the corresponding relationship between the number of CVD in the far field and different off-axis distance has been got, as well as, the influence of the beams parameters (waist width, spatial correlation length) and atmospheric turbulence parameters (structure constant, the inner scale of turbulence, the outer scale of turbulence) on the corresponding relationship has been stressed. The results obtained in this paper are useful for understanding the evolution behavior of the CVD in turbulence and for comparison with previous work.

* Corresponding author.
E-mail address: jinhongli@live.cn (J. Li).
[1] These authors contributed equally to this work and should be considered co-first authors.

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2. Theoretical formulation

Consider the CVD embedded in a Gaussian beam whose field at the initial plane $z=0$ in the rectangular coordinate system is written as [20]

$$E(\mathbf{s},0)=\left[\frac{(s_x-a)+i(s_y-b)}{w_0}\right]\left[\frac{(s_x+a)-i(s_y+b)}{w_0}\right]\exp\left(-\frac{s_x^2+s_y^2}{w_0^2}\right)$$
$$=\left[\frac{s_x^2+s_y^2-a^2-b^2+2i(as_y-bs_x)}{w_0^2}\right]\exp\left(-\frac{s_x^2+s_y^2}{w_0^2}\right), \tag{1}$$

where a and b are the off-axis distance parameters in the x direction and y direction, respectively. $\mathbf{s}\equiv(s_x,s_y)$ is the two-dimensional (2D) position vector, w_0 denotes the waist width of the Gaussian part.

By introducing a Schell-correlator [21], the cross-spectral density function of GSM beams with the CVD at the source plane $z=0$ is expressed as

$$\begin{aligned}W(\mathbf{s}_1,\mathbf{s}_2,0)&=\Big\{s_{1x}^2s_{2x}^2+s_{1x}^2s_{2y}^2+s_{2x}^2s_{1y}^2+s_{1y}^2s_{2y}^2-(a^2+b^2)\\&\quad(s_{1x}^2+s_{2x}^2+s_{1y}^2+s_{2y}^2)+(a^2+b^2)^2\\&+2i\Big[as_{1x}^2s_{2y}+as_{1y}^2s_{2y}-bs_{1x}^2s_{2x}-bs_{1y}^2s_{2x}+(a^2+b^2)\\&\quad(bs_{2x}-as_{2y})(bs_{2x}-as_{2y})\Big]\\&-2i\Big[as_{2x}^2s_{1y}+as_{2y}^2s_{1y}-bs_{2x}^2s_{1x}-bs_{2y}^2s_{1x}+(a^2+b^2)\\&\quad(bs_{1x}-as_{1y})\Big]\\&+4\Big[a^2s_{1y}s_{2y}-abs_{2x}s_{1y}-abs_{1x}s_{2y}+b^2s_{1x}s_{2x}\Big]\Big\}\\&\quad\exp\left(-\frac{s_1^2+s_2^2}{w_0^2}\right)\exp\left(\frac{-(\mathbf{s}_1-\mathbf{s}_2)^2}{2\sigma_0^2}\right),\end{aligned} \tag{2}$$

where $\mathbf{s}_i=(s_{ix},s_{iy})$ $(i=1,2)$ is the 2D position vector at the source plane $z=0$, * denotes the complex conjugate, and σ_0 denotes the spatial correlation length.

In accordance with the extended Huygens-Fresnel principle [22], the cross-spectral density function of GSM beams with the CVD propagating through atmospheric turbulence are given by

$$\begin{aligned}W(\rho_1,\rho_2,z)&=\left(\frac{k}{2z\pi}\right)^2\\&\iint d^2\mathbf{s}_1\iint d^2\mathbf{s}_2W(\mathbf{s}_1,\mathbf{s}_2,0)\exp\Big\{\\&-\frac{ik}{2z}\Big[(\rho_1-\mathbf{s}_1)^2-(\rho_2-\mathbf{s}_2)^2\Big]\Big\}\\&\times\Big\langle\exp\Big[\psi^*(\rho_1,\mathbf{s}_1)+\psi(\rho_2,\mathbf{s}_2)\Big]\Big\rangle,\end{aligned} \tag{3}$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ denote the position vector at the z plane, k is the wave number related to the wave length λ by $k=2\pi/\lambda$, $\langle\bullet\rangle$ denotes the average over the ensemble of the turbulence medium, it is worth mentioning that a quadratic approximation of Rytov's phase structure function [23] is used in Eq. (3), which can be written as

$$\Big\langle\exp\Big[\psi^*(\rho_1,\mathbf{s}_1)+\psi(\rho_2,\mathbf{s}_2)\Big]\Big\rangle=\exp\Big\{-T\Big[(\rho_1-\rho_2)^2+(\rho_1-\rho_2)(\mathbf{s}_1-\mathbf{s}_2)+(\mathbf{s}_1-\mathbf{s}_2)^2\Big]\Big\}, \tag{4}$$

where T is the strength of atmospheric turbulence and used to characterize the effect of the atmospheric turbulence [24]

$$T=\frac{\pi^2k^2z}{3}\int_0^\infty k^3\Phi_n(k)dk, \tag{5}$$

where $\Phi_n(k)$ is the spatial power spectrum of the refractive-index fluctuations of the turbulent medium, having, for the Von karman case, the form [22]

$$\Phi_n(k)\approx 0.033C_n^2\frac{\exp(-k^2/k_m^2)}{(k^2+k_0^2)^{11/6}}\quad(0\le k\le\infty). \tag{6}$$

Substituting Eq. (6) into Eq. (5), we obtain

$$T=0.0033\pi^2k^2z\;C_n^2\left\{-2k_0^{\frac{1}{3}}+\exp\left(\frac{k_0^2}{k_m^2}\right)k_m^{-\frac{5}{3}}\left[\frac{5}{3}k_m^2+2k_0^2\right]\Gamma\left(\frac{1}{6},\frac{k_0^2}{k_m^2}\right)\right\}, \tag{7}$$

where $k_0=1/L_0$, $k_m=5.92/l_0$, L_0 and l_0 are the outer and inner scales of atmospheric turbulence, respectively, and C_n^2 being the structure constant of atmospheric turbulence with unit $m^{-2/3}$.

To simplify the calculation, introducing two variables of integration $\boldsymbol{\mu}=(\mathbf{s}_1+\mathbf{s}_2)/2$, $\boldsymbol{\nu}=\mathbf{s}_1-\mathbf{s}_2$, and recalling integral formula [25]

$$\int x^n\exp(-px^2+2qx)dx=n!\sqrt{\frac{\pi}{p}}\left(\frac{q}{p}\right)^n\exp\left(\frac{q^2}{p}\right)\sum_{k=0}^{E[n/2]}\frac{1}{(n-2k)!k!}\left(\frac{p}{4q^2}\right)^k. \tag{8}$$

Substituting Eqs. (2), (4) and (8) into Eq. (3), we obtain the cross-spectral density function of GSM beams with the CVD propagating through atmospheric turbulence as follows

$$W(\rho_1,\rho_2,z)=\left(\frac{k}{2zw_0^2}\right)^2\exp\left[-\frac{ik}{2z}(\rho_1^2-\rho_2^2)\right]\exp[-T(\rho_1-\rho_2)^2]\;(M_1+M_2+iM_3+iM_4), \tag{9}$$

where

$$\begin{aligned}M_1&=\frac{1}{AC}\Bigg[\frac{2(b^2-a^2)(E_x^2-E_y^2)-8abE_xE_y+2}{C^2}+\frac{4(E_x^2+E_y^2)}{C^3}\\&+\frac{(E_x^2+E_y^2)^2}{C^4}+\frac{1}{4C}\left(\frac{1}{F}-\frac{1}{A}\right)\\&\times\left(1+\frac{E_x^2+E_y^2}{C}\right)-\frac{B_x^2+B_y^2}{16A^2C}+\frac{G_x^2+G_y^2}{4CF^2}+\frac{G_y^2E_x^2+G_x^2E_y^2}{2C^2F^2}\\&-\frac{B_x^2E_x^2+B_y^2E_y^2+4B_xB_yE_xE_y}{8A^2C^2}\\&+\frac{5k^2(E_x^2+E_y^2)}{8A^2C^3z^2}+\frac{k^2(E_x^4+E_y^4+4E_x^2E_y^2)}{8A^2C^4z^2}\\&+\frac{kE_y^2(aE_y-2bE_x)+kE_x^2(2aE_y-bE_x)}{AC^3z}\\&+\frac{5k(aE_y-bE_x)}{2AC^2z}+\frac{5k^2}{16A^2C^2z^2}+(a^2+b^2)^2\Bigg]\exp\left(\frac{B_x^2+B_y^2}{4A}+\frac{E_x^2+E_y^2}{C}\right),\end{aligned} \tag{10}$$

$$\begin{aligned}M_2&=\frac{w_0^2}{F}\Bigg[\frac{1}{16F^2}+\frac{G_x^2+G_y^2}{8F^3}+\frac{(G_x^2+G_y^2)^2}{32F^4}-\frac{(a^2+3b^2)}{8F}\left(1+\frac{2G_x^2}{F}\right)\\&-\frac{(3a^2+b^2)}{8F}\left(1+\frac{2G_y^2}{F}\right)\\&+\frac{abG_xG_y}{F^2}\Bigg]\exp\left[\frac{G_x^2+G_y^2}{F}+\frac{w_0^2(D_x^2+D_y^2)}{8}\right],\end{aligned} \tag{11}$$