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Iterative approach for zero-order term elimination in off-axis multiplex digital holography

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ABSTRACT

An iterative approach is proposed to eliminate the zero-order term from an off-axis multiplexed hologram that contains several sub-holograms. The zero-order components of each sub-hologram are effectively eliminated one by one using the proposed iterative procedure. Because of the reduction of the zero-order components in the frequency domain, enlarged filtering windows can be used to separate each of the +1 order components and improve the signal-to-noise ratio. The proposed method does not require prior knowledge of the object images, and only needs each of the reference wave intensities, which can be acquired before acquisition of the multiplexed hologram. The feasibility of the proposed approach is confirmed through mathematical deductions and numerical simulations, and the robustness of the proposed approach is verified using a practical multiplexed hologram.

1. Introduction

Digital holograms can be multiplexed by encoding the information from two or more holograms into a single hologram, which is called a multiplexed hologram [1]. Optical multiplexing is used to produce a single-frame hologram in which several sub-holograms are multiplexed to obtain various object properties simultaneously. Off-axis multiplex digital holography (OMDH), which is based on spatially angular multiplexing, has drawn increasing attention in recent years, and has been widely used in various real-time investigations, including ultrafast events [2], 3D imaging in full color [3], aperture synthesis [4], detected area enlargement [5], field of view enhancement [6,7], and dynamic measurements [8].

However, there have been no reports in the literature to date about zero-order elimination in real-time multiplex holography systems. In all the applications mentioned above, all object reconstruction operations have been performed using the spectrum filtering method [9]. In general, in an OMDH system, the angles between the pairs of object waves and reference waves must be very small to satisfy the sampling theorem because of charge-coupled device (CCD) pixel size limitations. The separation of the zero-order and +1 order components in the frequency domain becomes difficult because there are large overlap regions for both orders. The selection of an appropriate filter window [9] becomes a crucial problem, involving consideration of the tradeoff between noise removal and preservation of the image details. Some zero-order elimination methods exist for use with conventional digital holograms, which only contain fringes from one object wave and reference wave pair, such as numerical zero-order elimination methods [10,13], and phase-shift methods [11,12]. It is obvious that the phase-shift methods are not suitable for real-time applications because of the time requirements for mechanical movement. Recently, a numerical iterative approach has been presented in off-axis multiplex holography that eliminated the zero-order more efficiently, because an estimate of the object wave intensity was used to remove the zero-order components [13]. However, this method is not suitable for use with multiplexed holograms. Here, we present an improved iterative approach to eliminate the zero-order term of an off-axis multiplexed hologram that contains several sub-holograms, and both simulations and experimental results have been used to demonstrate its effectiveness. The zero-order components of each sub-hologram can be eliminated effectively one by one using the proposed iterative procedure. Because of the zero-order components elimination in the frequency domain, enlarged filtering windows can be used to separate each of the +1 order components with an accompanying improvement in the signal-to-noise ratio. The proposed method does not require prior knowledge of the object image and only requires each of the reference wave intensities, which can be acquired before acquisition of the multiplexed hologram. The feasibility of the proposed approach is confirmed through mathematical deductions and numerical simulations, and the robustness of the approach is verified using a practical multiplexed hologram.

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2. Theoretical analysis

In off-axis multiplex digital holography, the intensity of the multiplexed hologram is given by

$$I(x, y) = o_1 r_1^* + o_1^* r_1 + o_2 r_2^* + o_2^* r_2 + \cdots + o_N r_N^* + o_N^* r_N + |o_1|^2 + |o_2|^2 + \cdots + |o_N|^2 + |r_1|^2 + |r_2|^2 + \cdots + |r_N|^2 \quad (1)$$

where $o_1, o_2 \ldots o_N$ and $r_1, r_2 \ldots r_N$ denote the object and reference waves, respectively, * denotes the conjugate operator, $|o_1|^2, |o_2|^2, \ldots |o_N|^2$ and $|r_1|^2, |r_2|^2, \ldots |r_N|^2$ are the zero-order components that should be eliminated. The amplitudes of the reference waves can be acquired in advance. Therefore, the point to be addressed for elimination of the zero-order image is the elimination of the $|o_1|^2, |o_2|^2, \ldots |o_N|^2$ components.

By subtracting the $|r_1|^2, |r_2|^2, \ldots |r_N|^2$ terms from the multiplexed hologram, we obtain the following:

$$I(x, y) = o_1 r_1^* + o_1^* r_1 + o_2 r_2^* + o_2^* r_2 + \cdots + o_N r_N^* + o_N^* r_N + |o_1|^2 + |o_2|^2 + \cdots + |o_N|^2 \quad (2)$$

By then applying a Fourier transform to Eq. (2), the spectral domain can be expressed as:

$$\hat{I}_0(\omega_x, \omega_y) = \hat{o}_1 \otimes \hat{o}_1^*(\omega_x, \omega_y) + \hat{o}_2 \otimes \hat{o}_2^*(\omega_x, \omega_y) + \cdots + \hat{o}_N \otimes \hat{o}_N^*(\omega_x, \omega_y) + \hat{o}_1 \otimes r_1 \delta_1(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) + \hat{o}_1^* \otimes r_1 \delta_1(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) + \hat{o}_2 \otimes r_2 \delta_2(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) + \hat{o}_2^* \otimes r_2 \delta_2(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) + \cdots + \hat{o}_N \otimes r_N \delta_N(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) + \hat{o}_N^* \otimes r_N \delta_N(\omega_x + \omega_{0,x}, \omega_y + \omega_{0,y}) \quad (3)$$

where the symbol ⊗denotes the convolution operation, and $(\omega_{0,x}, \omega_{0,y})$ is the spatial carrier frequency of the reference wave.

A circular filter of an appropriate size [13] is then used to extract the first +1 order of the multiplexed hologram in the spectral domain. Using the inverse Fourier transform for the +1 order, Eq. (4) is then obtained.

$$\mathcal{F}^{-1}\{\hat{I}_0(\omega_x, \omega_y)\widehat{W}_1(\omega_x, \omega_y)\} = o_1 r_1 e^{i\varphi} + (|o_1|^2 + |o_2|^2 + \cdots + |o_N|^2) \otimes W_1(x, y) = o_1 r_1 e^{i\varphi} + O_{W1}(x, y) \quad (4)$$

$\widehat{W}_1(\omega_x, \omega_y)$ is a window function and is used to select one of the imaging terms. By calculating the autocorrelation of Eq. (4), and by then dividing it with the reference intensity, the object wave intensity estimator for the first iterative step of the first sub-hologram zero-order elimination process is obtained as:

$$|o_{est1,1}|^2 = \frac{1}{|r_1|^2} \times [o_1 r_1 e^{i\varphi} + O_{W1}(x, y)][o_1 r_1 e^{i\varphi} + O_{W1}(x, y)]^* = (|o_1|^2 + \frac{|O_{W1}|^2}{r_1^2} + \frac{o_1}{r_1} O_{W1}^* e^{i\varphi} + \frac{o_1^*}{r_1} O_{W1} e^{-i\varphi}) = |o_1|^2 + \varepsilon_{1,1}(x, y) \quad (5)$$

where $\varepsilon_{1,1}(x, y) = \frac{|O_{W1}|^2}{r_1^2} + \frac{o_1}{r_1} O_{W1}^* e^{i\varphi} + \frac{o_1^*}{r_1} O_{W1} e^{-i\varphi}$.

The imaging term $|o_{est1,1}|^2$ is then extracted from the hologram to give

$$I_{est1,1} = I_0(x, y) - |o_{est1,1}|^2 = o_1 r_1^* + o_1^* r_1 + o_2 r_2^* + o_2^* r_2 + \cdots + o_N r_N^* + o_N^* r_N + |o_2|^2 + \cdots + |o_N|^2 + \varepsilon_{1,1}(x, y) \quad (6)$$

Eq. (6) is the new iteration of the initial multiplexed hologram with the first iteration of the first time zero-order elimination process. The iteration operation is then repeated $(k_N - 1)$ times, and the zero-order of the first sub-hologram of the multiplexed hologram is eliminated, as shown in Eq. (7).

$$I_{est1,k_1} = I_0(x, y) - |o_{est1,k_1}|^2 = o_1 r_1^* + o_1^* r_1 + o_2 r_2^* + o_2^* r_2 + \cdots + o_N r_N^* + o_N^* r_N + |o_2|^2 + \cdots + |o_N|^2 + \varepsilon_{1,k_1}(x, y) \quad (7)$$

This iterative process is then repeated for the other sub-holograms one by one. Ultimately, the zero-order term of the multiplexed hologram is eliminated, as shown in Eq. (8):

$$I_{estN,k_N} = o_1 r_1^* + o_1^* r_1 + o_2 r_2^* + o_2^* r_2 + \cdots + o_N r_N^* + o_N^* r_N + \varepsilon_{1,k_1}(x, y) + \varepsilon_{2,k_2}(x, y) + \cdots + \varepsilon_{N,k_N}(x, y) \quad (8)$$

The iterative error for the first iterative process for zero-order elimination of the first sub-hologram is shown in Eq. (9):

$$\varepsilon_{1,1}(x, y) = |o_1|^2 - \left|o_{est1,1}\right| = \frac{|O_{W1}|^2}{r_1^2} + \frac{o_1}{r_1} O_{W1}^* e^{i\varphi} + \frac{o_1^*}{r_1} O_{W1} e^{-i\varphi} \approx r_1^2 \left(\frac{o_1 \sum_{a=1}^N o_a^2}{r_1^3} + \frac{\sum_{b=1}^N \sum_{c=1}^N o_b^2 o_c^2}{r_1^4} \right) \quad (9)$$

A general formula for the error that occurs after iterative step k_1 of the first sub-hologram zero-order elimination process is given as:

$$\varepsilon_{1,k_1}(x, y) = |o_1|^2 - |o_{est1,k_1}| \propto r_1^2 \sum_{j=3}^{2^{k_1+1}} O_1 \left(\frac{\sum_{a=1}^N \sum_{b=1}^N o_a^i o_b^{j-i}}{r_1^j} \right) i \in 0, 1, 2 \ldots j \quad (10)$$

After iterative step k_N of the Nth sub-hologram zero-order elimination process, the general formula for the error is obtained as follows:

$$\varepsilon_{N,k_N}(x, y) = |o_N|^2 - |o_{estN,k_N}| \propto r_N^2 \sum_{j=3}^{2^{k_N+1}} O_1 \left(\frac{\sum_{a=1}^N \sum_{b=1}^N o_a^i o_b^{j-i}}{r_N^j} \right) i \in 0, 1, 2 \ldots j \quad (11)$$

The sum error is obtained after the N sub-holograms zero-order elimination process as:

$$\varepsilon_{sum} = \varepsilon_{1,k_1}(x, y) + \varepsilon_{2,k_2}(x, y) + \cdots + \varepsilon_{N,k_N}(x, y) \quad (12)$$

Based on Eq. (11), if the reference amplitudes are stronger than the object amplitudes, then ε_{sum} is convergent.

3. Numerical simulation

The simulation parameters are set as follows: the wavelength of the laser source is 800 nm, the number of pixels of the recording device is 512*512, with each individual pixel having dimensions of 3.75 µm*3.75 µm. A simulation using the English alphabet with the Windings typeface and the English alphabet with the Symbol typeface is performed to demonstrate the effectiveness and the convergence of the proposed method.

The reconstruction results for the iterative method described above are shown in Fig. 1, and were obtained using three iterations for both iterative processes.

The convergence of the multiplexed hologram is described by Eqs. (9)–(12). The speed of convergence for the multiplexed hologram is shown in Fig. 2. From the curves shown in Fig. 2, five iterative steps are sufficient to eliminate the zero-order term, and the amount of zero-order noise retained in Fig. 1(c) is 0.18 %, which proves that the method proposed to eliminate the zero-order term of the multiplexed hologram is highly efficient. We also computed the time taken to perform three iterative steps for both iterative processes. The time taken is 0.106789 s, which is sufficient time for efficient elimination of the zero-order image of the multiplexed hologram (this test was performed using a PC with a main frequency of 3.40 GHz and MATLAB). The results show that the proposed method is effective and has potential for application to processing of real-time multiplexed holograms.

The ability of the iterative approach to reconstruct the phase