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Quantum mechanical treatment of parametric amplification in an absorptive nonlinear medium

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ABSTRACT

Generally, loss phenomena are known to affect the quantum properties of a light wave. This paper describes a quantum mechanical treatment of parametric amplification in an absorptive nonlinear medium. An expression of the quantum mechanical field operator in such a physical system is presented based on the Heisenberg equation, using which the quantum properties of traveling light suffering from medium absorption are quantitatively evaluated. Calculations using the obtained operator indicate that some degradation of noise performance is caused by the absorption. The influence of the absorption on the squeezing performance in phase-sensitive parametric amplification is also evaluated.

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1. Introduction

Optical parametric amplification [1–3] is known to inherently have low-noise properties, due to the fact that it does not include the population inversion parameter (or the noise factor), which degrades the noise performance in population inversion based amplification. Moreover, phase-sensitive parametric amplification, where signal and idler lights are degenerated, offers a quantum-limited noise figure of 0 dB, i.e., extra noise free amplification, owing to the degeneracy effect. Phase-sensitive amplification is also useful for generating amplitude-squeezed light in quantum optics. Besides, parametric amplification as well as other nonlinear phenomena is available for various optical signal processing [1–5].

The above low-noise properties of parametric amplification, including the 0-dB noise figure in phase-sensitive amplification, are theoretically derived based on quantum mechanics [6–10]. However, conventional quantum-mechanical studies on parametric amplification assume a lossless nonlinear medium, not taking the loss effect into account, whereas quantum properties are known to be affected by loss phenomena in general. This may be because nonlinear media usually employed for parametric amplification, e.g., highly nonlinear fibers, have a low propagation loss, and the loss effect on quantum noise is supposed to be negligible small. However, how small the loss effect is has not been quantitatively clarified, because of the lack of quantum mechanical treatment of parametric amplification including medium loss. Regarding quantum properties of parametric amplification suffering from propagation loss, one study on the squeezing effect in phase-sensitive amplification was conducted by employing a model where beam splitters representing loss phenomena were locally inserted along a lossless nonlinear medium [11]. However, quantum noise properties were not investigated. Other studies, where loss phenomena and parametric amplification were separately treated, were also conducted [12,13], for analyzing fiber transmission systems composed of transmission lines and parametric amplifiers, employing a phenomenological beam-splitter model for the loss effect.

With the above background, this paper presents a quantum mechanical treatment of parametric amplification in an absorptive nonlinear medium, in order to quantitatively clarify the loss effect on quantum noise in parametric amplification. The spatial evolution of the quantum-mechanical field operator, that simultaneously includes parametric amplification and medium absorption, is derived based on the Heisenberg equation. Then, using the spatial-evolved operator, quantum noise of a signal light parametrically amplified in an absorptive medium is qualitatively evaluated. The influence of the absorption on the squeezing performance in phase-sensitive parametric amplification is also investigated.

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2. Quantum mechanical treatment

2.1. Spatial evolution of field operators

In the Heisenberg picture, quantum properties are evaluated by calculating the time evolution of physical quantity operators (e.g., the annihilation operator for the light wave field) and then by averaging the time-evolved operators with respect to the initial state. For calculating the motion of a quantum-mechanical operator, the Heisenberg equation with a Hamiltonian for a concerned physical system is used. In order to evaluate parametric amplification in an absorptive medium in this way, we should calculate the Heisenberg equation with a Hamiltonian simultaneously including parametric interaction and absorption. However, this straightforward approach seems troublesome. In the present paper instead, we first consider parametric amplification and a loss phenomenon separately, and then, based on the considerations, express the spatial evolution of the field operator that simultaneously takes parametric interaction and absorption into account.

We assume parametric amplification where a signal light with frequency f_s is amplified and an idler light with frequency f_i is generated by a pump light with frequency f_p, satisfying $f_s+f_i=2f_p$. The Hamiltonian for this physical system is expressed as [10]

$$\hat{H}_{opa} = \hbar\omega_s\hat{a}_s^\dagger\hat{a}_s + \hbar\omega_i\hat{a}_i^\dagger\hat{a}_i + i\hbar(\chi\hat{a}_s^\dagger\hat{a}_i^\dagger - \chi^*\hat{a}_s\hat{a}_i), \tag{1}$$

where $\hat{a}_s$ and $\hat{a}_i$ are the field operators (annihilation operators) of the signal and idler lights, respectively; χ represents the interaction efficiency determined by the nonlinear coefficient and the pump light amplitude; $\hbar$ is the Plank's constant; and ω_s and ω_i are the angular frequencies of the signal and idler lights, respectively. From the Heisenberg equation with the above Hamiltonian, the spatial evolution of the signal and idler field operators is derived as [10]

$$\hat{a}_{s,i}(z_0+\Delta z) = \hat{a}_{s,i}(z_0)\cosh(g\Delta z) + e^{i\phi}\hat{a}_{i,s}^\dagger(z_0)\sinh(g\Delta z), \tag{2}$$

where g is the gain coefficient proportional to the pump power, ϕ is the phase determined by the pump light phase, and the phase-matching condition is assumed to be satisfied.

Regarding absorption, on the other hand, interactions with an ensemble of two-level systems are assumed to cause light attenuation. The Hamiltonian for such a physical system is expressed as [14]

$$\hat{H}_{abs} = \hbar\omega\hat{a}^\dagger\hat{a} + \sum_j \hbar\omega_m^{(j)}\hat{\pi}_j^\dagger\hat{\pi}_j + i\sum_j \hbar(\alpha_j\hat{\pi}_j^\dagger\hat{a} - \alpha_j^*\hat{\pi}_j\hat{a}^\dagger), \tag{3}$$

where $\hat{\pi}$ is the medium transition operator from the upper to lower energy states; α_j represents the interaction efficiency; $\hbar\omega_m^{(j)}$ is the energy difference between the upper and lower states in the medium; and subscript j labels each two-level system in the ensemble. From the Heisenberg equation with the above Hamiltonian, the spatial evolution of the field operator of traveling light in an absorptive medium is derived as [14]

$$\hat{a}(z_0 + \Delta z) = \hat{a}(z_0)\hat{b} + \hat{c}. \tag{4}$$

In this expression, $\hat{b}$ and $\hat{c}$ are the medium operators composed of $\hat{\pi}$. They have properties of $\langle\rangle\hat{b}\rangle=e^{-\alpha\Delta z/2}$; $\langle\hat{c}\rangle=\langle\hat{c}^\dagger\hat{c}\rangle=0$; and $\langle\hat{c}\hat{c}^\dagger\rangle=1-e^{-\alpha\Delta z/2}$, where $\langle\rangle$ denotes quantum-mechanical averaging over the medium state, and α is the spatial attenuation coefficient. Note that this expression is applicable to a local area (i.e., $\Delta z\ll 1$) [14]. Eq. (4) is equivalent to the beam-splitter model, such that $\hat{b}$ corresponds to the amplitude transmittance, and $\hat{c}$ corresponds to the noise field operator overlapped onto the field operator as a result of absorption.

By combining Eqs. (2) and (4), we can express the spatial evolution of the field operator in parametric amplification in an absorptive medium. For this, a classical expression for that phenomenon is helpful. The classical wave equation for that physical system is

$$\frac{dE_{s,i}}{dz} = -\frac{\alpha}{2}E_{s,i} + i\gamma(2\left|E_p\right|^2 + \left|E_{s,i}\right|^2 + 2\left|E_{i,s}\right|^2)E_{s,i} + i\gamma E_p^2E_{i,s}^*, \tag{5}$$

where E_s, E_i, and E_p are the signal, idler, and pump amplitudes, respectively, and γ is the nonlinear coefficient. Under assumptions that pump depletion is negligible and the phase-matching condition is satisfied, the solution of Eq. (5) is expressed as

$$E_{s,i}(z_0 + \Delta z) = \left\{E_{s,i}(z_0)\cosh(g\Delta z) + e^{i\phi}E_{i,s}^*(z_0)\sinh(g\Delta z)\right\}e^{-\alpha\Delta z/2}, \tag{6}$$

where parameters g, ϕ, and α have the same meanings as in Eqs. (2) and (4). Eq. (6) indicates that the spatial evolution of the classical light amplitude in parametric amplification in an absorptive medium is given by that in a lossless medium multiplied by the amplitude transmittance. We apply this classical expression to the spatial evolution of the field operators, based on the fact that the average of the field operator corresponds to the classical light amplitude. In addition, Eq. (4) suggests that the noise field operator is superimposed through the absorption process. With these considerations, we have the following expression for the spatial change of the field operator in parametric amplification in an absorptive medium:

$$\hat{a}_{s,i}(z_0 + \Delta z) = \left\{\hat{a}_{s,i}(z_0)\cosh(g\Delta z) + \hat{a}_{i,s}^\dagger(z_0)e^{i\phi}\sinh(g\Delta z)\right\}e^{-\alpha\Delta z/2} + \hat{c}_{s,i}. \tag{7}$$

with $\langle\hat{c}_{s,i}\rangle=\langle\hat{c}_{s,i}^\dagger\hat{c}_{s,i}\rangle=0$ and $\langle\hat{c}_{s,i}\hat{c}_{s,i}^\dagger\rangle=1-e^{-\alpha\Delta z/2}$. We evaluate quantum properties of parametric amplification with absorption, using this expression.

2.2. Output signal field operator

Eq. (7) expresses the spatial evolution in a local area and cannot be straightforwardly applied to the in-out relationship of the length of a nonlinear medium, because it is based on Eq. (4) that is applicable just to a local area. In order to obtain the operator at the output of the length of a nonlinear medium, we divide the light propagation distance in the medium into small segments, and apply Eq. (7) to each segment as

$$\hat{a}_{s,i}^{(k+1)} = \left\{\hat{a}_{s,i}^{(k)}\cosh(g_k\Delta z) + \hat{a}_{i,s}^{(k)\dagger}e^{i\phi}\sinh(g_k\Delta z)\right\}e^{-\alpha\Delta z/2} + \hat{c}_{s,i}^{(k)}, \tag{8}$$

where the superscript (k) denotes the operators in the kth segment, and the medium operator $\hat{c}_{s,i}^{(k)}$ satisfies $\langle\hat{c}_{s,i}^{(k)}\rangle=\langle\hat{c}_{s,i}^{(k)\dagger}\hat{c}_{s,i}^{(k)}\rangle=0$ and $\langle\hat{c}_{s,i}^{(k)}\hat{c}_{s,i}^{(k\prime)\dagger}\rangle=\{1-\exp(-\alpha\Delta z)\}\delta_{kk'}$. The gain coefficient g is also labeled by the segment number k because the pump power attenuates through propagation, while the loss coefficient α is constant throughout the medium. By iteratively applying the above equation to each successive segment of the whole propagation distance in the medium, we obtain the field operator at the medium output as

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