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Nonuniform optical sampling signal reconstruction algorithm

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ABSTRACT

optical pulse generated by mode-locked lasers has lower time jitter than electronics, which is the main advantage of optical sampling. However, by using multiplexer to further increase the repetition rate of optical pulse will cause nonuniformity in amplitude and time delay. Here we discuss the effect brought by this kind of static deviation, and propose the reconstruction algorithm in Nth-order nonuniform optical sampling. Experiment results show that it can effectively build the original signal.

1. Introduction

Digital signal processing toward high-speed has always been the hot topic in research. Continued advances of electronic method over the past few decades have made astonishing progress in data processing capability, containing sampling speed, bandwidth and so on. However, limited by clock jitter and inherent ambiguity in electronics [1], further advance of capability of electronic analog-to-digital conversion (EADC) has met the great challenge.

Mode locked lasers have been developed in recent years with time jitter of tens of femtosecond or even shorter [2], which is far below the electronics jitter. This key advantage enables the photonic method to overcome the limitation of EADC. Several kinds of method combing electronics and optics has been developed [3–6]. As the key device, the mode locked laser generally has a repetition frequency from megahertz to tens of gigahertz. To satisfy the requirement of higher sampling speed, optical time division multiplexing (OTDM) or wavelength division multiplexing (WDM) is often applied to increase the repetition frequency. In these systems, variable delay lines and attenuators were often used to precisely match the amplitude and time between paths [5,7,8]. This method not only reduces the system stability, but also increases the power loss, the difficulty of control and cost with increasing multiplexing paths. More power loss means less SNR, thus wasting the advantage of low time jitter. Apparently, design of fixed system with pre-calculation will bring better stability and feasibility for integration, as well as inevitable time and amplitude deviation. As a result, nonuniform sampling pulses are generated. In other words, nonuniform sampling decreases the problem in control, but transfers the difficulty to signal processing. The corresponding algorithm, known as nonuniform sampling signal reconstruction algorithm, is more complicated than traditional algorithm in uniform sampling.

Here we analyze the effect in sampling caused by the deviation when increasing the repetition frequency by multiplexing, and the related *N*th order nonuniform sampling signal reconstruction algorithm. We experiment on a three cascaded OTDM optical sampling system, and the result, which meanwhile compares with other interpolating methods, has demonstrated the validity of the algorithm.

2. Theoretical analysis

To achieve an optical sampling system with high bandwidth and sampling rate, the first step is to multiplex the original optical pulses. There are several kinds of multiplexing in optical paths, but for all types, a single optical pulse propagating through the system will generate sampling pulse trains, which can be expressed as

$$y_p(t) = \frac{1}{N}\sum_{i=1}^{N}\left[a_i \cdot \delta\left(t - \Delta_i\right)\right] \tag{2.1}$$

where N is the quantity of optical pulses after multiplexing, and a_i and Δ_i represent the amplitude and time delay of the ith pulse. The pulse is taken as Dirac delta function for simplicity. Usually, N is equal to multiplexing paths, and a_i and Δ_i are respectively the amplitude and time delay of the ith path. This kind of system is easy to operate and control, but difficulty and cost will rise with increasing paths. Here we propose and analyze the model of cascaded OTDM, which can increase the repetition rate much higher than common WDM or OTDM and is used in our experiment in the next section, as illustrated in Fig. 1.

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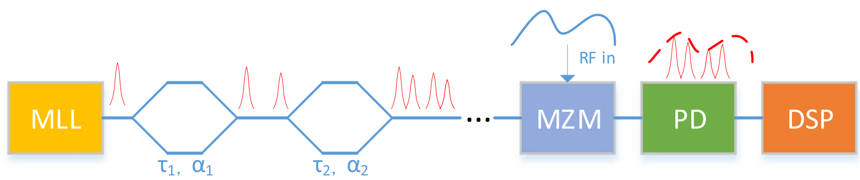

Fig. 1. A simple model of nonuniform sampling by cascaded OTDM. MLL: mode locked laser, MZM: Mach-Zehnder modulator, PD: photo diode; DSP: digital signal processor.

In each OTDM, the time delay and power dissipation between two paths caused by fiber, couplers and others can be combined as one in one path. We express them as the single normalized amplitude coefficient α and time delay τ, then the output after first OTDM can be written as

$$y_1(t) = \frac{1}{2}y_0(t) + \frac{1}{2}\alpha_1 y_0(t-\tau_1) = \frac{1}{2}\delta(t) + \alpha_1\frac{1}{2}\delta(t-\tau_1) \tag{2.2}$$

Similarly, the relationship of the output between the $(k-1)$th and kth OTDM is

$$y_k(t) = \frac{1}{2}y_{k-1}(t) + \frac{1}{2}\alpha_k y_{k-1}(t-\tau_k) \tag{2.3}$$

The output after kth output can be obtained by calculating (2.3) iteratively. The result will contains 2^k terms. By expressing the time delay τ as phase angle in complex form, the output can be simply written as

$$y_k(t) = \frac{1}{N}\prod_{m=1}^{k}(1+\alpha_m e^{j\tau_m}) \tag{2.4}$$

Where N is equal to 2^k. Expanding $y_k(t)$ will produce N terms, which indicates that cascaded OTDM generates N pulses. The amplitude and phase of the ith term $(i \le N)$ represent the amplitude and time delay of the ith pulse respectively, which is correspond with a_i and Δ_i in (2.1) (The so-obtained pulses generally are not arranged by time. By convention they can be sorted after calculation).

Hence, for a mode-locked laser with repetition frequency f_{laser} and period $T = 1/f_{laser}$, the average sampling rate is $f_s = Nf_{laser}$, and the optical sampling streams can be expressed as

$$y(t) = \frac{1}{N}\sum_{i=1}^{N}\sum_{p=-\infty}^{\infty} a_i\delta(t-pT-\Delta_i) = \frac{1}{N}\sum_{i=1}^{N}\sum_{p=-\infty}^{\infty} a_i\delta\left(t-p\frac{N}{f_s}-\Delta_i\right) \tag{2.5}$$

It is shown, that the changes of amplitude and time delay in each OTDM result in relative deviations in amplitude and time delay between pulses. Without precise control, these static deviations may be large or small, and reconstruction can meet large distortion if they are simply treated as common sampling errors. In the assumption of linear response, amplitude deviations a_i can be simply removed before interpolating, or be considered as coefficients in interpolating process equivalently. However in practice the linearity is not perfect, and low power is more susceptible to noise and amplitude jitter. So it is suggested to ensure equal amplitudes in the system design. For simplicity, the amplitude deviations won't be considered in the reconstruction algorithm. The deviations of time delay result in different sample time, implying different signal informations. That's what makes the Nth-order nonuniform sampling reconstruction algorithm more complicated. Considering the typical applications, we assume the input signal is lowpass below Nyquist sampling rate. A detailed analysis of Nth-order bandpass sampling has been discussed in [9].

Leaving out the amplitude deviations, the pulses sampling the signal $x_a(t)$ produce

$$x_a^{\delta N}(t) = \frac{x_a(t)}{N}\sum_{i=1}^{N}\sum_{p=-\infty}^{\infty}\delta\left(t-p\frac{N}{f_s}-\Delta_i\right) \tag{2.6}$$

The result can be taken as sum of N sampling steams with identical period $T = N/f_s$. We assume that the ith stream sampling $x_a(t)$ produces $x_i(pT) = x_a(pT+\Delta_i)$ with sample time $t = pT + \Delta_i$, and through interpolating function $g_a^{(i)}(t)$ for the ith sampling stream we can obtain a continuous signal in (2.7)

$$y_a^{(i)}(t) = \sum_{p=-\infty}^{\infty} x_i(pT)g_a^{(i)}(t-pT-\Delta_i) \tag{2.7}$$

where $i = 1, 2, \ldots, N$. Its Fourier transformation is calculated as [10]

$$Y_a^{(i)}(F) = G_a^{(i)}(F)\frac{1}{T}\sum_{k=-\infty}^{\infty} X_a(F-\frac{k}{T})e^{-j2\pi\frac{k}{T}\Delta_i} \tag{2.8}$$

The reconstructed signal is obtained by summing N interpolated streams, which is

$$y_a(t) = \sum_{i=1}^{N} y_a^{(i)}(t) = \sum_{i=1}^{N}\sum_{p=-\infty}^{\infty} x_i(pT)g_a^{(i)}(t-pT-\Delta_i) \tag{2.9}$$

And its Fourier transformation is

$$Y_a(F) = \sum_{i=1}^{N} G_a^{(i)}(F)\frac{1}{T}\sum_{k=-\infty}^{\infty} X_a(F-\frac{k}{T})e^{-j2\pi\frac{k}{T}\Delta_i} \tag{2.10}$$

For Nth-order sampling, define $T = 1/B$, and (2.10) can be written as

$$Y_a(F) = \sum_{i=1}^{N} G_a^{(i)}(F)B\sum_{k=-\infty}^{\infty} X_a(F-kB)e^{-j2\pi kB\Delta_i} \tag{2.11}$$

We assume the signal is lowpass with two-sided bandwidth NB. It should be noted that in an Nth-order optical sampling system totally or partly consisting of cascaded OTDM, N is always even. This is very important when analyzing the aliasing in frequency domain because situation would be more complicated with odd N.

Aliasing can be discussed in different regions in frequency domain. According to (2.11), the spectrum is a sum of infinite replica of X_a with the same frequency interval B. For the convenience of analysis, the bandwidth of original signal $-NB \le F \le NB$ can be divided into positive and negative regions. As shown in Fig. 2, for the positive region (marked with thick border), it is aliased by other positive replicas (Fig. 2(a)) and negative replicas (Fig. 2(b)), and situations are different in each sub frequency range of B. For the range $mB \le F \le (m+1)B$, $m = 0, 1, \ldots, N/2-1$ (red region), there are one original spectrum ($k = 0$ with thick border), $N/2-1$ aliasing caused by positive replicas ($k = m, m-1, \ldots, m-N/2+1$ and $k \ne 0$ in Fig. 2(a)), and $N/2$ aliasing caused by negative replica ($k = m+1, m+2, \ldots, m+N/2$ in Fig. 2(b)).

To reconstruct the original signal, the aliasing should be removed. This can be realized according to the principle that the sum of original spectrum equals 1 and sum of all other aliasing equals 0. Therefore we can construct N equations as follows