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Three-dimensional analysis of free-space light propagation based on quantum mechanical scattering theory of light

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A B S T R A C T

In this paper, a free-space light propagation analysis between 3-dimensional (3-D) volumetric spaces is proposed. In contrast to conventional scalar diffraction, the proposed theory is based on quantum mechanical scattering providing a general volumetric analysis for the free-space light propagation. Assuming a plane wave light incidence, we obtained a new analytic formula for 3-D volumetric convolution, which provided a transfer function in a closed form used for caculating the electric fields at the observation points. The proposed method was consistent with the conventional numerical methods for a 2-dimensional aperture and can be further applied to exact calculation of diffraction fields from 3-D surfaces, providing a compact reconstruction algorithm for 3-D images in a computer generated hologram.

1. Introduction

Low-energy elastic photon scattering has not been in the research spotlight, despite recent prominent achievements in quantum optics, mainly due to the following reasons. First, the wave function and Hamiltonian for a single photon which fulfills both Maxwell's equations and quantum electrodynamics are still controversial [1–4]. Second, most light scattering issues have been well explained in the classical regime without introducing the quantum nature of light [5]. Consequently, the scattering or diffraction of light without involving absorption and emission processes has been treated in the classical physics regime.

Conventional classical methods for the light propagation analysis of diffracted waves, such as Huygens convolution method and the angular spectrum method [6], are commonly based upon the first Rayleigh–Sommerfeld formula, which originated from the mathematical relation known as the Green's second identity [7]. These classical scalar diffraction methods can be described as a plane-to-plane transformation between the source plane and the observation plane, as schematically illustrated in Fig. 1(a). Recently, the volume-to-volume transformation in three-dimensional (3-D) space is being intensively investigated along with rapid development of 3-D displays and holographic applications [8]. A 3-D volumetric transformation requires a systematic algorithm that can represent the precise relationship between the source space and the observation space as illustrated in Fig. 1(b). A 3-D angular spectrum method has been recently reported [8], but its theoretical background still stems from the first Rayleigh–Sommerfeld equation, result in the limitation of applications to a planar aperture.

In this paper, we report a new calculation method for 3-D free-space light propagation based on the newly-developed scattering theory starting from quantum-mechanical description of a single photon, enabling systematic volume-to-volume transformation for an arbitrary 3-D object. The clear definition of a wave-function and Hamiltonian of a single photon provides a theoretical background for applying perturbation assumption to the scattering theory of a photon. We successfully obtained an analytic description of quantum mechanical scattering theory for photons and consistently related it with 3-D light diffraction, for the first time to the best knowledge of the authors.

There are distinctive light scattering processes in nature such as Thompson scattering [9], Mie scattering [10], and Rayleigh scattering, with different light-matter interaction origins. Since our main concern is only on the macroscopic behavior of a photon in free-space, we do not focus on the origin of light-matter interaction, but only on the propagation behavior of a photon itself within the observation volume in the far field.

2. 3-D light propagation formula using the quantum mechanical scattering theory of a single photon

Quantum mechanical description of space and momentum was

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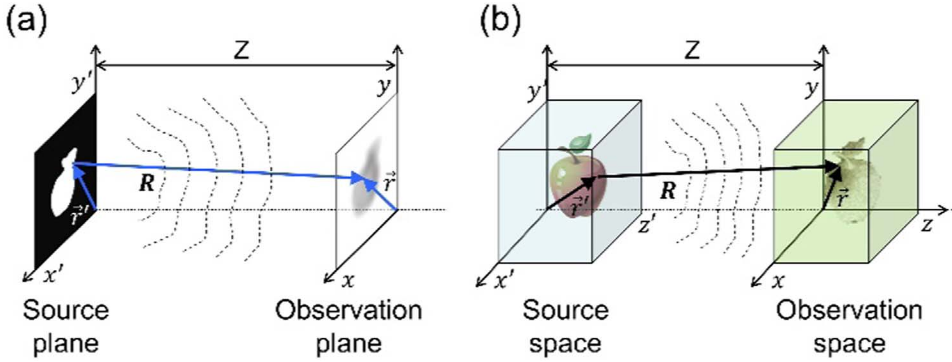

Fig. 1. The geometries for (a) the conventional approach of the diffraction theory and (b) the proposed 3-dimensional transformation between diffraction fields.

found to provide a consistent theoretical basis for the 3-D Fourier transform between the source and observation spaces, which further simplified the numerical analyses. In this study, we used the time-independent perturbation theory [11] of quantum mechanical scattering. The Hamiltonian of a perturbed photon state is given by $\hat{\mathbf{H}} = \hat{\mathbf{H}}_0 + \hat{\mathbf{V}}$, where $\hat{\mathbf{H}}_0$ represents the unperturbed Hamiltonian of a photon state and the perturbation term $\left(\hat{\mathbf{V}}\right)$ represents the light-matter interaction. In order to bridge the gap between the electromagnetic diffraction and the quantum mechanical scattering, we started with definition of operators and the wave function of a single photon. According to recent studies [3,4], a six-component wave-function and a Hamiltonian for a photon, given by $\hat{\mathbf{H}}_0 = ic\hat{\mathbf{p}}\times$ in the momentum space, showed a good physical consistency between the Maxwell equations and the quantum electrodynamics in describing the quantum mechanical properties of light. However, we found that a three-component wave-functions comprised of a Riemann- Silberstein (R-S) state vector $\left(|\boldsymbol{\psi}\rangle = |\mathbf{E}\rangle + ic|\mathbf{B}\rangle\right)$ [1,2,12] are more appropriate and convenient to develop our 3-D scattering theory. Assuming a propagating electro-magnetic wave satisfying $\hat{\mathbf{k}} \times \mathbf{E} = \mathbf{B}$, the energy eigenvalue for a single photon can be expressed in R-S vectors as:

$$\hat{\mathbf{H}}_0|\boldsymbol{\psi}\rangle = ic\hbar\hat{\mathbf{k}} \times \left(|\mathbf{E}\rangle + ic|\mathbf{B}\rangle\right) = c\hbar\left(ick|\mathbf{B}\rangle + k|\mathbf{E}\rangle\right) = c\left|\vec{\mathbf{p}}\right||\boldsymbol{\psi}\rangle \tag{1}$$

Starting from the Lippmann–Schwinger equation [13], the perturbed wave function can be described as:

$$\left\langle \vec{r} \middle| \psi^{\pm} \right\rangle = \left\langle \vec{r} \middle| \phi \right\rangle + \left\langle \vec{r} \left| \frac{\hat{V}}{E - \hat{H}_0 \pm i\varepsilon} \right| \psi^{\pm} \right\rangle \tag{2}$$

Here, we will focus only on the scattered wave, the 2nd term in the right hand side of Eq. (2), and neglect the un-scattered wave, the 1st term in the right hand side of Eq. (2), as in general quantum mechanical approaches. Using the 1st Born approximation [11], the outgoing wave function, denoted by $\left\langle \vec{r} \middle| \psi^{+} \right\rangle$ in the observation space, can be written in terms of the source space, $\vec{r}\,'$, as below.

$$\left\langle \vec{r} \middle| \psi \right\rangle = \left\langle \vec{r} \left| \frac{\hat{V}}{E - \hat{H}_0 + i\varepsilon} \right| \phi \right\rangle = \int d^3r' \left\langle \vec{r} \left| \frac{1}{E - \hat{H}_0 + i\varepsilon} \right| \vec{r}\,' \right\rangle \left\langle \vec{r}\,' \middle| \hat{V} \middle| \phi \right\rangle \tag{3}$$

By using the quantum-mechanical descriptions for space-momentum and assuming a photon-matter interaction potential of V_0 [See Appendix A], the output wave-function $\psi_o\left(\vec{r}\right)$is given by:

$$\psi_o\left(\vec{r}\right) = -\frac{k_0}{2\pi}\cdot\frac{V_0}{\hbar c}\iiint_V d^3r' \frac{e^{ik_0\left|\vec{r}-\vec{r}\,'\right|}}{\left|\vec{r} - \vec{r}\,'\right|}\psi_m\left(\vec{r}\,'\right) \tag{4}$$

By using the definition of R-S vectors in Eq. (1) and equating the real and imaginary parts, Eq. (4) resulted in equations for either the electric or the magnetic field vector. Electric field components are obtained as:

$$E_o(x, y, z) = -\frac{ik_0\tilde{K}}{2\pi} \times \iiint dx'dy'dz' \frac{e^{ik_0\sqrt{(x-x')^2+(y-y')^2+(Z+z-z')^2}}}{\sqrt{(x-x')^2 + (y-y')^2 + (Z+z-z')^2}} E_m(x', y', z') \tag{5}$$

Here, we introduced a new constant $\tilde{K} \equiv V_0/i\hbar c$ which has the same dimensions as the wave number. It is worthwhile to compare Eq. (5) with the 2-dimensional (2-D) diffraction formula based on the Huygens-Fresnel principle [6]:

$$E_o(x, y; Z) = -\frac{ik_0}{2\pi}\iint_S dx'dy' \frac{e^{ik_0R}}{R} E_i(x', y'; 0) \tag{6}$$

It is evident that Eq. (5) is a 3-dimensional analogue of Eq. (6) and the 3-D formula, Eq. (5), is consistent with the 2-D formula, if we assume the photon-matter interaction potential V_0 is related only with the geometrical structure along the z-direction. This assumption is generally valid if we consider that our observation space is macroscopically distant from the source space, and therefore we can only consider paraxial components parallel to z-axis.

Using the convolution theorem, Eq. (5) can be further expressed as a 3-dimensional convolution.

$$E_o(x, y, z) = (2\pi)^{3/2}\cdot\mathcal{F}_3^{-1}\left\{\mathcal{F}_3\left(E_m(x', y', z')\right)\cdot T\left(k_x, k_y, k_z\right)\right\} \tag{7}$$

$$T\left(k_x, k_y, k_z\right) \equiv \mathcal{F}_3\left(\frac{-ik_0\tilde{K}}{2\pi}\frac{e^{ik_0\sqrt{x'^2+y'^2+(Z+z')^2}}}{\sqrt{x'^2 + y'^2 + (Z+z')^2}}\right) \tag{8}$$

Here, $\mathcal{F}_3(\bullet)$ and $\mathcal{F}_3^{-1}(\bullet)$ represent the 3-dimensional Fourier and inverse Fourier transform. In this equation, the first factor in the inverse Fourier transform is a 3-dimensional spectrum of the input field distribution. The second factor $T\,(k_x,k_y,k_z)$, defined in Eq. (8), acts as a 3-dimensional transfer function.

For a plane wave incident along the z-axis, the modulated electric field $E_m(x', y', z')$ includes the modulation function $m(x', y', z')$ and the highly oscillating term $e^{ik_0z'}$ of the input electric field as follows.

$$E_m(x', y', z') = E_0 m(x', y', z')e^{ik_0z'} \tag{9}$$