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Comparison of finite element and transfer matrix methods for numerical investigation of surface plasmon waveguides

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ABSTRACT

In this paper, we investigate Surface Plasmon Polaritons (SPPs) in the visible regime at a metal/dielectric interface within two different waveguide structures, the first is a Photonic Crystal Fiber where the Full Vector Finite Element Method (FVFEM) is used and the second is a slab waveguide where the transfer matrix method (TMM) is used. Knowing the diversities between the two methods in terms of speed, simplicity, and scope of application, computation is implemented with respect to wavelength and metal layer thickness in order to analyze and compare the performances of the two methods. Simulation results show that the TMM can be a good approximation for the FVFEM and that SPPs behave more like modes propagating in a semi infinite metal/dielectric structure as metal thickness increases from about 150 nm.

1. Introduction

Surface Plasmon Polaritons (SPPs) are bound electromagnetic waves that propagate in metal surfaces and exponentially decay in the transverse direction, they are the result of coupling between electron charges oscillations (Plasmon) and photons [1] and satisfy Maxwell's equations. In the last years, SPPs have been extensively investigated through various research fields, studying the dispersion properties of SPP waveguides is very important due to its usage in the design of different optoelectronic devices such as modulators [2], filters [3], sensors [4], etc. Some works used instead of metals, other materials for the generation of SPPs due to their tunable propagation properties, such as doped semiconductors [5] or the recently discovered graphene which has promising aptitude in the field of optics [2,3,6]. In sensing applications, chemical sensing and biosensing in particular, the Surface Plasmon Resonance (SPR) technique is widely used to excite SPPs. Previous works have employed SPR using prisms to channel light towards metal/dielectric interfaces [7,8]. In Subsequent works Optical Fibers started to replace prisms [9,10], the experimental setup was remarkably simplified compared to Kretschmann prism [11]. Optical Fibers provide a small size, possibility of remote sensing, enhanced sensitivity and biocompatibility [10]. In recent years Photonic Crystal Fibers (PCF) also gained access to SPR research field [12,13]. Compared to conventional optical fibers, PCFs offer a better control of the mode profile and more flexibility in the design due to their various geometrical parameters, they can be made of one material and can guide light in a hollow core (HCPCF) (Photonic Band Gap) [14]. An HCPCF based SPR sensor was recently reported to achieve a very good sensitivity [15].

The Full Vector Finite Element Method (FVFEM) is one of the most famous numerical tools handling 2D or 3D inhomogeneous and anisotropic waveguides, it allows to solve a large class of partial differential equations without any limitation by geometry [16]. It was employed in many works to simulate PCF based SPR sensors in the visible [4], near infrared [17,18] and mid infrared [19] regimes. In the terahertz (THz) regime, Themistos et al. [20] have used the FEM to study surface plasmon modes in a metal clad dielectric waveguide.

In SPR applications, the FVFEM is used to calculate propagating modes at a given wavelength range in order to predict the resonance point where the waveguide and SPP modes are phase matched.

The FVFEM can be sometimes a consuming method in terms of time and memory, in particular when dealing with relatively thin layers that require a high number of elements to discretize them accurately or with materials having complex dielectric constant.

The transfer matrix method (TMM) is an analytical method exploited to study simple and homogenous models. Basically, it is used to calculate reflection and transmission coefficients in a multilayer stack [21]. However, the TMM can also be employed to solve waveguide problems [5,21,22]. It was used in some works to predict SPR sensors responses, calculations and experimental

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results were compared, the TMM was then reported to be a good modelization tool for both Kretschmann prism [23] and optical fiber [11] based sensors. In Ref. [5] the TMM was used to analyze a dielectric loaded SPP waveguide in the THz and near IR regimes, it was compared to the FEM and proved to be a rapid and accurate method. In the mid infrared regime Ref. [24] employed the TMM for the study of a graphene/dielectric multilayer structure to realize Fano resonances.

The TMM we use here is a fast and simple analytical method, but its usage is limited to 1D models.

In this work, an analogy is made between the finite element and transfer matrix methods by computing SPP modes supported by PCF and slab plasmonic waveguides respectively. In the first waveguide which is gold coated PCF, shown in Fig. 1, consisting of one ring of air holes with a hole diameter d and pitch Λ, we use the FVFEM in the analysis. In the slab plasmonic waveguide shown in Fig. 2, we use the TMM. In addition, the semi infinite metal/dielectric structure is also studied for comparison as shown in Fig. 3. The thickness of gold layer is denoted t and ε_1, ε_2 and ε_3 are dielectric constants of silica, gold and the outer medium respectively and they are used for the three structures.

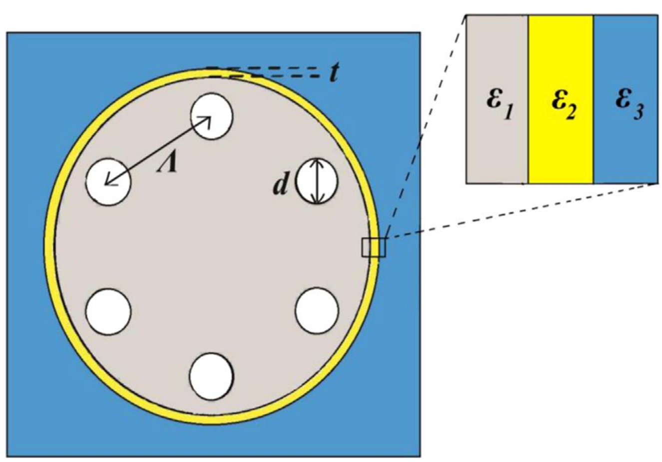

Fig. 1. Plasmonic PCF with one ring of air holes. The hole diameter and the pitch are d and Λ respectively, ε_1, ε_2 and ε_3 are dielectric constants of silica, gold, and the outer medium respectively, t is the thickness of the gold layer.

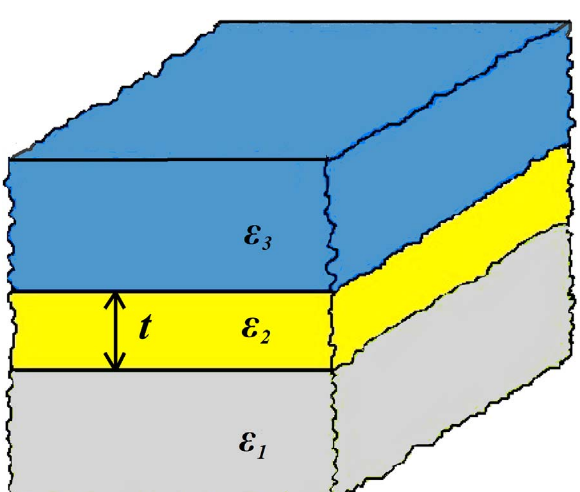

Fig. 2. Plasmonic Slab waveguide structure, ε_1, ε_2 and ε_3 are dielectric constants of silica, gold, and the outer medium respectively, t is the thickness of the gold layer.

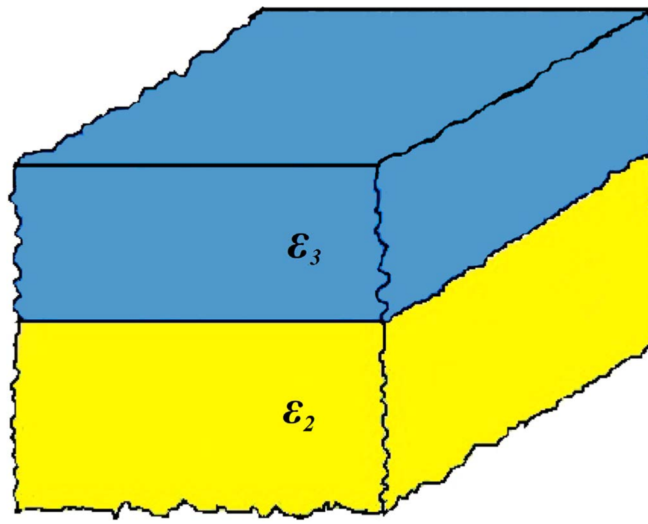

Fig. 3. Semi infinite gold/dielectric structure, ε_2 and ε_3 are dielectric constants of gold and the outer medium respectively.

2. Theory of simulation methods

2.1. Finite element method

The Full Vector Finite element method FVFEM is a powerful approach for analyzing optical waveguides problems where propagating modes are the main interest [25]. We implement the FVFEM in a computer code as a numerical tool to investigate propagation characteristics of the waveguide shown in Fig. 1 by solving the following vector wave equation derived from Maxwell's equations [25,26]:

$$\nabla \times (\nabla \times \boldsymbol{E}) - k_0^2 \varepsilon_i \boldsymbol{E} = 0, \tag{1}$$

where $\boldsymbol{E}$ is the electric field, $k_0(=2\pi/\lambda)$ is the free space wavenumber, λ is the free space wavelength and ε_i is the dielectric constant.

The plasmonic PCF is discretized into hybrid edge/nodal triangular elements. The advantage of such elements is that tangential continuity of the electric field is imposed by default at the interfaces between different media since all involved field components are tangential [25].

Applying the FVFEM on Eq. (1) yields the following eigenvalue equation [26]:

$$[\mathrm{K}]\{\boldsymbol{E}\} - \beta^2 [\mathrm{M}]\{\boldsymbol{E}\} = 0, \tag{2}$$

[K] and [M] are the global matrices and β is the propagation constant. Eq. (2) is numerically solved to calculate SPP modes supported by the waveguide.

2.2. Transfer matrix method

The transfer matrix method is applied to multilayer structures where the incident, reflected and transmitted light beams are related by a global matrix called the Transfer Matrix, resulting from the product of elementary matrices characterizing each layer and interface in the system [21,22].

A stack of parallel layers having different thicknesses t_i and dielectric constants ε_i, is illustrated in Fig. 4. A_1, B_1 and A_N are the incident, reflected and transmitted waves respectively, they are related by the following equation: