Optics Communications 382 (2017) 580–584

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Entanglement of two optically driven quantum dots mediated by phonons in nanomechanical resonator ☆

Yong He *, Meiping Jiang *

*School of Mathematics and Physics, Changzhou University, Changzhou 213164, China*

ARTICLE INFO

*Article history:*
Received 29 June 2016
Received in revised form
17 August 2016
Accepted 18 August 2016

*Keywords:*
Quantum dots
Cavity quantum electrodynamics
Quantum entanglement

ABSTRACT

The exciton–phonon coupling between an optically driven quantum dot (QD) and a mechanical resonator can be described by Jaynes–Cummings model under a certain condition, revealing phonon absorption and emission. When two optically driven QDs share a common phonon mode, it shows the phonon-mediated coupling between the QDs. Based on the effective master equation for the reduced density matrix of the two QDs, the temporal evolution of each state and the concurrence (quantum entanglement) between them are studied. The results suggest that the stationary concurrence depends strongly on the resonator temperature. The non-negligible entanglement in the hybrid system is advantaged to develop solid-state quantum information processing.



## 1. Introduction

Recently nanotechnology has allowed semiconductor quantum dots (QDs) embedded into a nanomechanical resonator for fabricating new hybrid systems [1–4]. The lowest-order flexural vibration mode of nanomechanical resonator modifies the energy of the electronic states of QD via the deformation potential coupling [2–5]. In QD-mechanical resonator hybrid system the strain-mediated coupling between the exciton of QD and the phonon mode of nanomechanical resonator, the exciton–phonon coupling, has been investigated both theoretically and experimentally [1–6]. Some schemes for laser cooling of phonon mode to its ground state have been presented [5,7–10], which are important for the potential applications of mechanical resonator in high precision detection of mass [11] and mechanical displacement [12,13]. As the exciton of QD is driven by a red-detuned laser, the cooling of phonon mode occurs by absorption of both phonon and photon to produce exciton. In the presence of a blue-detuned laser, however, the heating of phonon mode corresponds to the phonon emission of QD. The phonon absorption and emission processes in optically driven QD can be studied for achieving the phonon laser [14–16] and quantum information processing based on the exciton–phonon coupling [17,18].

Quantum entanglement between two spatially separated QDs is one of the most intriguing phenomena, which has attracted the increasing attention despite being challenging experimentally. In principle, as two QDs interact with each other by means of energy transfer, the entangled states between them can be generated. There are various hybrid systems where quantum entanglement appears. In cavity quantum electrodynamics system in which two QDs are placed in a common cavity, the entangled states can be generated via cavity photon transfer [19–21]. Surface plasmon field in metal nanowire [22,23] or nanoparticle [24,25] is an ideal quantum bus to entangle two QDs. Phonon mode of nanomechanical resonator plays an important role on the generation of the stationary entanglement between two cavity modes [26]. If two spatially separated QDs driven by laser share a phonon mode of nanomechanical resonator, they may interact with each other by means of phonon transfer [27] (phonon absorption and emission [28]), so that it is possible to create quantum entanglement. In this paper, we firstly investigate the strain-mediated coupling between a mechanical resonator and a QD driven by a laser, revealing the phonon absorption and emission of the optically driven QD under certain condition. In the weak coupling regime, two optically driven QDs interact with each other by means of phonon transfer. To reveal quantum entanglement between them, we demonstrate the temporal evolution of concurrence.

## 2. Phonon absorption and emission of optically driven QD

We consider a hybrid system consisting of a semiconductor quantum dot (QD) embedded in a nanomechanical resonator. At low temperatures, QD can be modeled as a two-level system,

☆Fully documented templates are available in the elsarticle package on http://www.ctan.org/tex-archive/macros/latex/contrib/elsarticleCTAN.

* Corresponding authors.
*E-mail addresses:* hey@cczu.edu.cn (Y. He), mpjiang@cczu.edu.cn (M. Jiang).

http://dx.doi.org/10.1016/j.optcom.2016.08.045

which consists of the ground state $|g\rangle$ and the first excited state $|e\rangle$. Meanwhile, an external optical field with frequency $\omega$ is exploited to drive the QD. The Hamiltonian can be described by $(\hbar = 1)$ $H_{QD} = \frac{\omega_{ex}}{2}\sigma_z + \frac{\Omega}{2}(\tilde{\sigma}e^{i\omega t} + H.\,c.\,)$, where $\omega_{ex}$ is the exciton frequency, $\tilde{\sigma} = |g\rangle\langle e|$ is the lowering operator, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\omega$ and $\Omega$ are the frequency of the optical driving field and the Rabi frequency, respectively. For the mechanical resonator, the lowest-energy resonance corresponds to the fundamental flexural mode that will constitute the resonator mode. The Hamiltonian of this mechanical mode is given by $H_m = \omega_m b^{\dagger}b$, where $\omega_m$ is the vibrational frequencies of the mechanical mode, $b^{\dagger}(b)$ represents the creation (annihilation) operator of the mechanical mode. Furthermore, since the flexion induces the extension and compression in the structure, the longitudinal strain will modify the energy of the electronic states of the QD via the deformation potential coupling. Then the Hamiltonian of the mechanical resonator coupled to QD is described by [2,3] $H_{int} = g_m\sigma_z(b^{\dagger} + b)$, where $g_m = \frac{\partial\omega_{ex}}{\partial x}x_{ZPF}$ represents the hybrid vacuum coupling rate between them, $x_{ZPF} = \sqrt{\hbar/(2m_{eff}\omega_m)}$ is the zero-point fluctuation amplitude, The parameter $\frac{\partial\omega_{ex}}{\partial x}$ is the exciton frequency shift per vibrational displacement quantum, which can be estimated by optical measurements of the exciton frequency of the QD and the vibrational displacement of the mechanical resonator [2,3].

In the rotating frame at the external field frequency $\omega$, the total Hamiltonian can be written as

$$H = \omega_m b^{\dagger}b + \frac{\Delta}{2}\sigma_{z,R} + \frac{\Omega}{2}\sigma_{x,R} + g_m\sigma_{z,R}(b^{\dagger} + b), \tag{1}$$

where $\Delta = \omega_{ex} - \omega$, $\sigma_{z,R} = |e_R\rangle\langle e_R| - |g_R\rangle\langle g_R|$, $\sigma_{x,R} = |g_R\rangle\langle e_R| + |e_R\rangle\langle g_R|$. $|g_R\rangle$ comprises the ground state and the incoming laser photons. $|e_R\rangle$ comprises the excited state and laser field with one photon less [28]. Two Pauli operators $\sigma_{z,R}$ and $\sigma_{x,R}$ can be transformed to two new Pauli operators $\sigma^z$ and $\sigma^x$ and by means of a rotation matrix, i.e.,

$$\begin{pmatrix}\sigma^z\\ \sigma^x\end{pmatrix} = \begin{pmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}\sigma_{z,R}\\ \sigma_{x,R}\end{pmatrix}, \tag{2}$$

where $\cos(\theta) = \frac{\Delta}{\omega_0}$, $\sin(\theta) = \frac{\Omega}{\omega_0}$, and $\omega_0 = \sqrt{\Delta^2 + \Omega^2}$ is the effective Rabi frequency. Here, we choose two superposition states $|+\rangle = \sin(\frac{\theta}{2})|g_R\rangle + \cos(\frac{\theta}{2})|e_R\rangle$, $|-\rangle = \cos(\frac{\theta}{2})|g_R\rangle - \sin(\frac{\theta}{2})|e_R\rangle$ (see Fig. 1) as the basic of the QD' states, so that

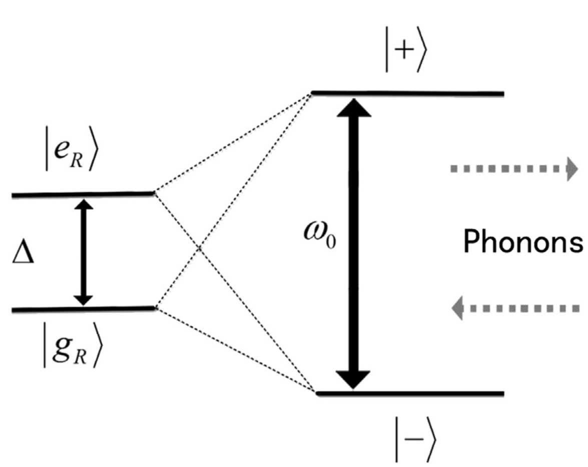


**Fig. 1.** Energy levels of optically driven QD to absorb and emit phonons. $|g_R\rangle$ and $|e_R\rangle$ denote the ground state and exciton state in the rotating frame, respectively. $|+\rangle = \sin(\frac{\theta}{2})|g_R\rangle + \cos(\frac{\theta}{2})|e_R\rangle$, $|-\rangle = \cos(\frac{\theta}{2})|g_R\rangle - \sin(\frac{\theta}{2})|e_R\rangle$, $\cos(\theta) = \frac{\Delta}{\omega_0}$, $\sin(\theta) = \frac{\Omega}{\omega_0}$, $\Omega$ represents the Rabi frequency, $\Delta$ denotes the exciton-field frequency detuning and $\omega_0 = \sqrt{\Delta^2 + \Omega^2}$ is the effective Rabi frequency.

$\sigma^z = |+\rangle\langle +| - |-\rangle\langle -|$, $\sigma^x = |+\rangle\langle -| + |-\rangle\langle +|$. The Hamiltonian can be rewritten as

$$H = \omega_m b^{\dagger}b + \frac{\omega_0}{2}\sigma^z + (G_0\sigma^z + G\sigma^x)(b^{\dagger} + b), \tag{3}$$

where $G_0 = g_m\cos(\theta)$, $G = -g_m\sin(\theta)$.

If $|\frac{\Delta}{\Omega}| \ll 1$, $\cos(\theta) \simeq 0$ and $\sin(\theta) \simeq 1$. Under the condition of $|\omega_m + \omega_0| \gg |\omega_m - \omega_0|$, one can neglect the fast oscillate terms $|+\rangle\langle -|b^{\dagger}e^{i(\omega_m+\omega_0)t}$ and $|-\rangle\langle +|be^{-i(\omega_m+\omega_0)t}$ (the rotating-wave approximation) so that the coupling between the excitons and the phonons can be described by Jaynes–Cummings model [10,13,29,30],

$$H = \omega_m b^{\dagger}b + \frac{\omega_0}{2}\sigma^z + G(b\sigma^{\dagger} + b^{\dagger}\sigma), \tag{4}$$

where $\sigma = |-\rangle\langle +|$. The coupling between the excitons and the phonons suggests the phonon absorption and emission of the optically driven QD.

The couplings between the hybrid system and the environment can be treated within the Born–Markov approximation for eliminating the bath phonon modes and the radiation field [5]. For the QDs, the environment includes the LA phonons in the crystal lattice of the QD, the other (lower-order) phonon modes in the mechanical resonator and various kinds of the radiation fields, to induce the exciton dissipation from the first excited state to the ground state whose rate can be obtained by observing the linewidth of the absorption spectrum of the QD. The bath phonon modes can also cause the thermoelastic damping of the fundamental flexural mode via the phonon–phonon interaction. The master equation of the hybrid system is given by

$$\partial_t\rho = i[\rho, H] + \zeta(\rho), \tag{5}$$

where the dissipative term $\zeta(\rho) = \kappa D[\tilde{\sigma}]\rho + \gamma_m(n_{th} + 1)D[b]\rho$ $+\gamma_m n_{th}D[b^{\dagger}]\rho$, $\tilde{\sigma} = \frac{1}{2}(\sigma^z + \sigma - \sigma^{\dagger})$, $\kappa$ and $\gamma_m$ are the dissipative rates of the exciton in the QD and the fundamental flexural mode in the mechanical resonator, respectively, $D[o] = o\rho o^{\dagger} - (o^{\dagger}o\rho + \rho o^{\dagger}o)/2$ denotes the Liouvillian in Lindblad form for operator $o$, and $n_{th} = [e^{\hbar\omega_m/(k_B T)} - 1]^{-1}$ corresponds to the thermal phonon number at the environmental temperature $T$, $k_B$ is the Boltzmann constant.

## 3. Phonon transfer of two driven QDs

Next, we consider two sufficiently separated QDs embedded in an identical mechanical resonator in the presence of an external field $E$, as shown in Fig. 2a. Each QD is coupled to the mechanical resonator via the deformation potential coupling. The flexion of the mechanical resonator causes the compression (Fig. 2b) and extension (Fig. 2d) in the structure which are accompanied by the changes of the exciton frequency. Since the distance between the two QDs $L \sim \mu$ m is much larger than the size of the QDs so that it is valid to ignore the dipole–dipole interaction between them [31,32]. The Hamiltonian can be written as

$$H_T = \omega_m b^{\dagger}b + \sum_{i=1,2}\frac{\omega_{0,i}}{2}\sigma_i^z + G_i(b\sigma_i^{\dagger} + b^{\dagger}\sigma_i), \tag{6}$$

where $\sigma_i = |-_i\rangle\langle +_i|$, $\omega_{0,i}$ is the effective Rabi frequency of $i$th QD in the presence of an external driving field. $G_i$ represents the vacuum coupling strength between the mechanical resonator and $i$th QD. The master equation of the total system is given by $\partial_t\rho = i[\rho, H_T] + \zeta_T(\rho)$, where the dissipative term $\zeta_T(\rho) = \sum_{i=1,2}\kappa_i D[\tilde{\sigma}_i]\rho + \gamma_m(n_{th} + 1)D[b]\rho + \gamma_m n_{th}D[b^{\dagger}]\rho$. To study the phonon transfer between the two QDs we have to obtain the reduced density matrix of the two QDs. First, we take a time-independent unity transformation $U$ on the density matrix $\rho$, where