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Phase shifts extraction based on time-domain orthogonal character of phase-shifting interferograms

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Based on the time-domain orthogonal character of different pixel intensity variation of phase-shifting interferograms, a novel non-iterative algorithm is proposed to achieve the phase shifts in random phase-shifting interferometry. Due to there is no requirement for the fringe number of phase-shifting interferograms, the proposed algorithm can work well even in the case that the fringe number of interferogram is less than one, which is a difficult problem in interferometry. Moreover, only two one-dimensional vectors, achieved from the average intensity of several pixels of interferogram, are enough to perform the phase shifts extraction, the proposed algorithm reveals rapid processing speed. Specially, compared with the conventional phase shifts extraction algorithms, the proposed algorithm does not need to perform the pixel-pixel calculation or the iterative calculation, so its processing speed is greatly improved. Both the simulation and the experiment demonstrate the outstanding performance of the proposed algorithm.

1. Introduction

Phase-shifting interferometry (PSI) [1], known as a high accuracy phase measurement technique [2], has been widely utilized in various fields, such as optical surface measurement[3], 3-D profile measurement [4], pressure and stress testing [5], digital holography [6], etc. To date, phase shifts extraction and phase retrieval are still two important research contents in PSI. Usually, phase retrieval is achieved from a sequence of phase-shifting interferograms with known phase shifts, such as the equal step-length or the fixed step-length phase-shifting algorithms [7,8], the least squares algorithms [9,10]. Consequently, the accuracy of phase retrieval is greatly affected by the phase-shifts error induced by the environment vibration and the non-linear of phase shifter. To address this, some algorithms are proposed to perform phase shifts extraction and phase retrieval from phase-shifting interferograms with random phase-shifts. First, an iterative algorithm based on the least squares is proposed [11], and then an advanced iterative algorithm (AIA) is developed [12], in which only three-frame phase-shifting interferograms are enough to achieve phase shifts extraction and phase retrieval. However, to ensure the measuring accuracy, it is required that the fringe number of interferogram should be more than one. Based on the principle component analysis (PCA), Vargas et al. achieve high accuracy phase shifts extraction and phase retrieval [13,14]. However, since the background is eliminated with the temporal averaging operation, this algorithm requires that the phase shifts should be uniformly distributed in an integer period [15]. By introducing Doppler Effect, a time-domain Fourier transform based phase retrieval algorithm is proposed. Though this algorithm possesses high accuracy, but its experiment setup is complicated and the processing is time-consuming [16]. Based on searching for the intensity maximum and minimum of each pixel of a sequence of phase-shifting interferograms, the corresponding background and modulation amplitude of interferogram can be determined [17], and then the wrapped phase with sign ambiguity can be retrieved by an arccosine algorithm easily (ACA) [18]. After that, several similar algorithms are also developed [19,20]. However, in these ACA algorithms, due to the background and the modulation amplitude of the interferogram being calculated by pixel by pixel, the processing is also time-consuming. To improve the processing speed, by introducing the notion of Euclidean matrix norm (EMN) [21], our group proposes a fast and simple phase shifts extraction algorithm with arccosine function. Subsequently, by introducing the notion of 1-norm, a novel phase shifts extraction algorithm is proposed (IN) [22], in which no requirement for the fringe number in the interferogram, but if the phase shifts is randomly changed, the extracted phase shifts might be not accurate due to the sign ambiguity of arccosine algorithm.

In this study, by introducing the time-domain orthogonal

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character (TOC) of different pixel intensity of phase-shifting interferograms, a novel non-iterative phase shifts extraction algorithm is proposed. Following, we will introduce the principle of the proposed algorithm, and then present the simulation and experimental results.

2. Principle

In random PSI, one-frame phase-shifting interferograms can be described as

$$i_{k,r} = a_r + b_r \cos\left[\varphi_r + \theta_k\right] \tag{1}$$

where $k(k=1, 2, \cdots, K)$ and $r(r=1, 2, \cdots, m, \cdots, n, \cdots, R)$ respectively represent the sequence number of phase-shifting interferograms and the pixel coordinate; a_r, b_r and ϕ_r denote the background, the modulation amplitude and the measured phase, respectively; θ_k denotes the phase shifts of the kth frame phase-shifting interferogram.

For simplicity, the intensity distribution of the rth pixel in K-frame phase-shifting interferogram can be described as a $1 \times K$ dimensional vector

$$I_r = \left\{ i_{1,r} \quad i_{2,r} \quad \cdots \quad i_{k,r} \quad \cdots \quad i_{K,r} \right\} \tag{2}$$

Similarly, the intensity distribution of arbitrary two pixels $(r=m, n)$ can be respectively expressed as

$$I_m = \left\{ i_{1,m} \quad i_{2,m} \quad \cdots \quad i_{k,m} \quad \cdots \quad i_{K,m} \right\} \tag{3}$$

$$I_n = \left\{ i_{1,n} \quad i_{2,n} \quad \cdots \quad i_{k,n} \quad \cdots \quad i_{K,n} \right\} \tag{4}$$

If the phase shifts is large enough, the intensity maximum and minimum of I_m and I_n can be denoted as i_{mmax}, i_{nmax} and i_{mmin}, i_{nmin}, respectively. Thus, after removing the background of interferogram, I_m and I_n can be respectively normalized as

$$\tilde{u} = \left\{ \tilde{u}_1, \tilde{u}_2, \cdots, \tilde{u}_k, \cdots, \tilde{u}_K \right\} \tag{5}$$

$$\tilde{v} = \left\{ \tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_k, \cdots, \tilde{v}_K \right\} \tag{6}$$

where

$$\tilde{u}_k = \frac{i_{k,m} - (i_{m\,max} + i_{m\,min})/2}{(i_{m\,max} - i_{m\,min})/2} = \cos(\varphi_m + \theta_k) \tag{7}$$

$$\tilde{v}_k = \frac{i_{k,n} - (i_{n\,max} + i_{n\,min})/2}{(i_{n\,max} - i_{n\,min})/2} = \cos(\varphi_n + \theta_k) \tag{8}$$

And then we define $\Theta_k=\phi_m+\theta_k$, and the phase difference Δ between two chosen pixels is that $\Delta=\phi_n-\phi_m$. In the case that $i_{k,m}=i_{mmax}$, there is the relationship $k=k_0(1 \le k_0 \le K)$. And then $\tilde{u}_{k_0}=1$, the value of $\tilde{v}_{k_0}$ is equal to $\cos(\Delta)$. Thus, Δ can be determined by

$$\Delta = \arccos(\tilde{v}_{k_0}) \tag{9}$$

If $\tilde{u} \neq \pm \tilde{v}$, showing two vectors are not parallel, thus $\tilde{u}$ and $\tilde{v}$ can be respectively orthonormalized as

$$\hat{u}_k = \tilde{u}_k = \cos(\Theta_k) \tag{10}$$

$$\hat{v}_k = \frac{\tilde{v}_k - \cos\Delta \cdot \tilde{u}_k}{-\sin\Delta} = sin(\Theta_k) \tag{11}$$

From Eqs. (9) and (10), we can achieve

$$\Theta_k = \arctan\left(\frac{\hat{v}_k}{\hat{u}_k}\right) \tag{12}$$

Then the phase shifts can be expressed as

$$\theta_k = \Theta_k - \Theta_1 \tag{13}$$

Clearly, by introducing the TOC of different pixel intensity of phase-shifting interferograms, we can achieve the phase shifts easily. However, due to the result calculated from single pixel intensity is easily affected by noise, in this study, we will employ the regional average intensity of interferogram instead of single pixel intensity of interferogram. If G denotes the total pixel number of calculated region of the kth frame phase-shifting interferogram, its average intensity can be expressed as

$$\bar{i}_{k,G} = \frac{1}{G}\sum_{g=1}^{G} i_{k,g} = A_G + B_G \cos(\varphi_G + \theta_k) \tag{14}$$

where

$$\begin{cases} A_G = \frac{1}{G}\sum_{g=1}^{G} a_g \\ B_G = \frac{1}{G}\left[\left(\sum_{g=1}^{G} b_g \cos\varphi_g\right)^2 + \left(\sum_{g=1}^{G} b_g \sin\varphi_g\right)^2\right]^{1/2} \\ \varphi_G = \arctan\left[-\left(\sum_{g=1}^{G} b_g \sin\varphi_g\right)\Big/\left(\sum_{g=1}^{G} b_g \cos\varphi_g\right)\right] \end{cases} \tag{15}$$

Consequently, if the regional average intensity of interferogram is employed to perform phase shifts extraction instead of the single pixel intensity of interferogram, the above phase shifts extraction method described as Eqs. (4)–(13) also should work well. In general, there are two criterions to choose the G region in Eqs. (14) and (15). One is that the fringe distribution of two chosen regions should be different. The other one is the size of chosen region, in principle, the larger size of chosen region, the less effect of noise for phase shifts extraction. Our preliminary calculation shows that when the pixel number of chosen region is more than 1000, the effect of noise can be ignored.

Clearly, due to the spatial sampling frequency is greatly larger more than the temporal phase-shifting sampling frequency, the accuracy of phase shifts extraction with the spatial sampling method will be significantly higher than with the temporal sampling method. That is to say, using the above two suppositions, we can achieve the accurate phase shifts extraction through using the spatial domain algorithm instead of the temporal domain algorithm. Of course, there is a main drawback the above algorithm: a lot of phase-shifting interferograms (at least 20-frame) are needed for background subtraction. However, by using the average algorithm expressed as Eqs. (14) and (15), the accuracy of phase shifts extraction will be improved.

3. Simulation

Numerical simulation is employed to verify the validity of the proposed TOC method. A sequence of phase-shifting interference patterns with size of 300×300 pixels are generated according to Eq. (1), in which the background intensity and the modulation amplitude are respectively set as $a(x,y)=10\exp[-0.25(x^2+y^2)]+120$ and $b(x, y)=100\exp[-0.25(x^2+y^2)]$, where $-1.5\text{ mm} \le x, y \le 1.5\text{ mm}$; the theoretical phase is set as $\phi(x, y)=3.3(x^2+y^2)$, the total number of phase-shifting interference patterns is equal to 30, and the phase shifts of interference pattern is defined as $\theta_k=2\pi(k-1)/28+\eta_k$, where η_k denotes the error induced by phase shifter and its range is set as -0.30 to 0.30. In addition, Gaussian additive noise with noise level of 5% is added to interference pattern, in