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Propagation properties of electromagnetic rectangular multi-Gaussian Schell-model beams in oceanic turbulence

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ABSTRACT

A model of electromagnetic rectangular multi-Gaussian Schell-model (ERMGSM) beams is introduced. Its analytic expression for the elements of the cross-spectral density matrix of such beams passing through oceanic turbulence is derived. It is shown that the rectangular shape of the ERMGSM beams holds a small distance on propagation in oceanic turbulence. The spectral density, the degree of coherence and the degree of polarization of ERMGSM beams are also studied in detail. The results will be helpful for underwater communication by using ERMGSM beams.

1. Introduction

In recent years a lot of random beams [1–7] have been introduced and explored, among them a class named rectangular multi-Gaussian Schell-model (RMGSM) beams [8] has been shown that they can generate a far-field intensity distribution with rectangular symmetry. Moreover, the intensity distribution keeps shape-invariant through the far field. Compared with multi-Gaussian Schell-model beams, the far-field intensity distribution shape of RMGSM beams is adjustable by changing the r.m.s. correlation widths along the x and y directions and the number of terms in the summation of the multi-Gaussian functions. Recently, the propagation properties of RMGSM beams in atmospheric turbulence have been investigated in [9] with scalar representations. Atmospheric turbulence results in random change of the beam direction [10,11] and additional beam spreading [12,13]. It is well known that there is another medium called oceanic turbulence which is very important in practical applications. It was found that the behavior of the propagation properties of the beams traveling in these two media is qualitatively different.

In this paper, we investigate the propagation of RMGSM beams in oceanic turbulence and extend its scalar model to the vectorial case. Because the polarization of beams is a fundamental property so it is worth to be studied. The oceanic turbulence will affect the direction of polarization and the polarization also has an influence on the beam wander induced by the oceanic turbulence. Here we only consider the case of clean-water oceanic turbulence which ignores the affection of suspended particles, air bubbles and so on. As is well known, the refractive index fluctuations of oceanic turbulence depend on temperature fluctuations and salinity fluctuations [14]. The temperature and salinity spectra have two similar profiles but the combined power spectrum is complex. Luckily a manageable analytic model considering two factors has been proposed [15]. By applying this analytic model and the extended Huygens–Fresnel principle, we derive an analytic expression of electromagnetic rectangular multi-Gaussian Schell-model (ERMGSM) beams passing through oceanic turbulence. Furthermore, we will study the average intensity, the degree of coherence and the degree of polarization of ERMGSM beams on propagation.

2. Electromagnetic rectangular multi-Gaussian Schell-model beams

In order to extend a scalar RMGSM beam to the electromagnetic case, we should derive the beam condition first. Using the method described in [16], we consider its element of the cross-spectral density matrix located in the plane $z = 0$ whose form is

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}'_1, \omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}'_2, \omega)}\,\mu_{ij}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1, \omega),\quad (i, j = x, y), \tag{1}$$

where $\boldsymbol{\rho}'_1 = (x'_1, y'_1)$, $\boldsymbol{\rho}'_2 = (x'_2, y'_2)$ are two positions, $s_i^{(0)}$ is the spectral density of the ith component and $\mu_{ij}^{(0)}$ represents the correlation coefficient. Assume that the spectral density has

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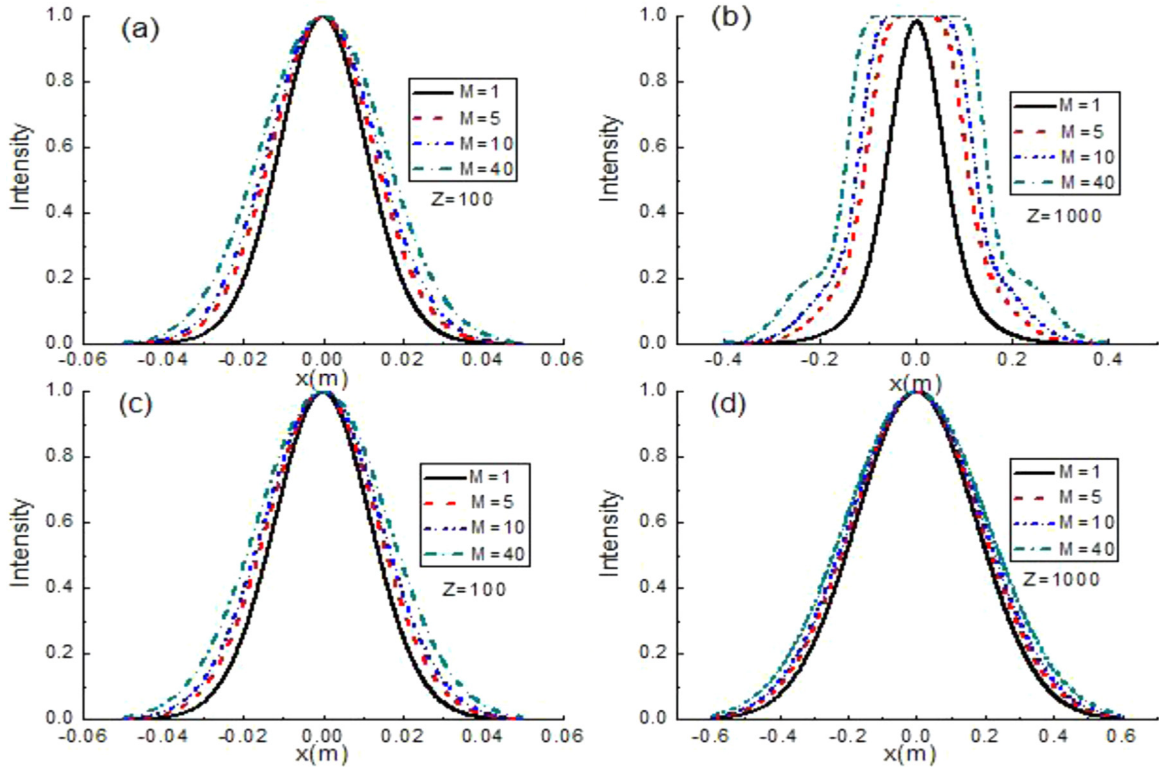

Fig. 1. Normalized intensity of the ERMGSM beams passing in free-space (a,b) and through oceanic turbulence (c,d) with different M.

the form

$$S_i^{(0)}(\boldsymbol{\rho}', \omega) = A_i^2 \exp\left(-\frac{\boldsymbol{\rho}'^2}{4\sigma_i^2}\right), \tag{2}$$

and the correlation coefficient of ERMGSM beams has the form

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = \frac{B_{ij}}{C_1 C_2} \times \sum_{m=1}^{M_1} \frac{(-1)^{m-1}}{\sqrt{m}} \binom{M_1}{m} \exp\left[-\frac{(x'_1 - x'_2)^2}{2m\delta_{ij}^2}\right] \times \sum_{m=1}^{M_2} \frac{(-1)^{m-1}}{\sqrt{m}} \binom{M_2}{m} \exp\left[-\frac{(y'_1 - y'_2)^2}{2m\delta_{ij}^2}\right], \tag{3}$$

where B_{ij} is the single-point correlation coefficient, δ_{ij} is the r.m.s. width of the correlation, $C_1 = \sum_{m=1}^{M_1} \frac{(-1)^{m-1}}{\sqrt{m}} \binom{M_1}{m}$ and $C_2 = \sum_{m=1}^{M_2} \frac{(-1)^{m-1}}{\sqrt{m}} \binom{M_2}{m}$ denote the normalization factors, A_i and σ_i are the amplitude and the r.m.s. width of the electric field, and $(\frac{M_1}{m})$ stands for the binomial coefficient. For simplicity, we consider $\sigma_i = \sigma_j = \sigma$. On substituting from Eqs. (2) and (3) into Eq. (1), one can obtain the elements of the cross-spectral density matrix of ERMGSM beams as

$$W_{ij}^{(0)}(x'_1, y'_1, x'_2, y'_2, \omega) = \frac{A_i A_j B_{ij}}{C_1 C_2} \exp\left(-\frac{x_1'^2 + x_2'^2 + y_1'^2 + y_2'^2}{4\sigma^2}\right) \times \sum_{m=1}^{M_1} \binom{M_1}{m} \frac{(-1)^{m-1}}{\sqrt{m}} \exp\left[-\frac{(x'_1 - x'_2)^2}{2m\delta_{ij}^2}\right] \times \sum_{m=1}^{M_2} \binom{M_2}{m} \frac{(-1)^{m-1}}{\sqrt{m}} \exp\left[-\frac{(y'_1 - y'_2)^2}{2m\delta_{ij}^2}\right]. \tag{4}$$

We know that the summation indices M_1 and M_2 can control the flat part and the sharpness of the x and y components, respectively. It is known that the correlation matrix must be quasi-Hermitian [3], thus we can obtain the condition that

$$B_{xx} = B_{yy} = 1, \ |B_{xy}| = |B_{yx}|, \ \delta_{xy} = \delta_{yx}. \tag{5}$$

By employing the result in [17], we will get an equivalent equation for the non-negative definiteness:

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = \int p_{ij}(\boldsymbol{\nu}) H_i^*(\boldsymbol{\rho}'_1, \boldsymbol{\nu}) H_j(\boldsymbol{\rho}'_2, \boldsymbol{\nu}) d\boldsymbol{\nu}, \tag{6}$$

here $p_{ij}(\boldsymbol{\nu})$ is non-negative and $H_j(\boldsymbol{\rho}'_2, \boldsymbol{\nu})$ is arbitrary. As to the ERMGSM beams, we have

$$H_i(\boldsymbol{\rho}', \boldsymbol{\nu}) = A_i \exp(-\frac{\boldsymbol{\rho}'^2}{4\sigma^2}) exp(-i\boldsymbol{\nu}\cdot\boldsymbol{\rho}') \tag{7}$$

and

$$p_{ij}(\nu) = \frac{B_{ij}\delta_{ij}^2}{C_1 C_2} \sum_{m=1}^{M_1} (-1)^{m-1} \binom{M_1}{m} \exp\left(-\frac{m\delta_{ij}^2 \nu^2}{2}\right) \times \sum_{m=1}^{M_2} (-1)^{m-1} \binom{M_2}{m} \exp\left(-\frac{m\delta_{ij}^2 \nu^2}{2}\right). \tag{8}$$

In order to satisfy the requirement described in [18], we must have

$$p_{xx}(\boldsymbol{\nu}) p_{yy}(\boldsymbol{\nu}) - p_{xy}(\boldsymbol{\nu}) p_{yx}(\boldsymbol{\nu}) \geq 0. \tag{9}$$

On substituting from Eq. (8) into Eq. (9) and performing some simplifications, we can obtain that