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Instantaneous frequency measurement by in-fiber 0.5th order fractional differentiation

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ABSTRACT

We experimentally demonstrate the possibility to retrieve the instantaneous frequency profile of a given temporal light pulse by in-fiber fractional order differentiation of 0.5th-order. The signal's temporal instantaneous frequency profile is obtained by simple dividing two temporal intensity profiles, namely the intensities of the input and output pulses of a spectrally-shifted fractional order differentiation. The results are supported by the experimental measurement of the instantaneous frequency profile of a mode-locked laser.

1. Introduction

Non-integer or fractional order differentiation dates back to the birth of the theory of differential calculus of integer order [1]. In the photonic domain, as far as we know, the first device capable of calculating the fractional order differentiation on the complex envelope of an incoming optical waveform was proposed in 2008 [2]. Since then, different proposals were presented to perform this task including: an asymmetrical π phase-shifted fiber Bragg grating [3], long-period grating [4,5], tilted fiber Bragg grating [6], silicon-on-isolator micro-ring resonators [7,8], and electrically assisted Mach–Zehnder interferometer [9]. The possibility to tune the fractional order of differentiation within some range is present in some of these devices [6–9]. Although is clear that a noticeable effort was done in the development of new photonic fractional order differentiator devices with better capabilities (such as fractional order tuning), little progress was achieved demonstrating some advantage in the use of these devices for a specific task.

On the other hand, due to its importance in the performance of fiber-optic communication systems today, new solutions are demanded for the instantaneous frequency monitoring of optical waveforms. There are renowned techniques able to perform this task, such as the frequency-resolved optical gating (FROG) [10,11], the spectral phase interferometry for direct electric field reconstruction (SPIDER) [12,13], and the multi-photon intra-pulse interference phase scan (MIIPS) [14]. However, they are typically best suited for short high intensity pulses well in the femtosecond regime, being of more limited application for broader optical pulses, i.e. from a few ps to well into the ns regime. Thus, new techniques have been proposed to retrieve the instantaneous frequency profile for longer optical temporal waveforms. In addition, the in-fiber solutions in which we are especially interested might be more practical for optical fiber systems. In References [15,16], a direct phase recovery technique was proposed from temporal intensity measurements at the input and output of a linear optical filter. However, precise knowledge of the filter's impulse response is necessary in amplitude and phase. More recently, a direct method for phase recovery based on the use of the transport of intensity equation was introduced, where two temporal intensity profiles at the input and output of a linear dispersive device are required [17]. On the other hand, in Ref. [18] it was shown that a spectrally shifted differentiator can be used to retrieve the phase profile of a given temporal optical waveform. However, the proposed algorithm also needs the numerical calculation of the first-order derivative of the modulus of the input signal. As expected, this numerical procedure is very sensitive to the presence of noise.

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Finally, in Ref. [19] it was demonstrated, through fractional calculus tools, that a very simple expression relates the instantaneous frequency, with the temporal intensities of the temporal waveform under test and either its 0.5th order fractional integration or differentiation. The developed theory was supported by numerical simulations; however no experimental realization was provided.

In this work we provide experimental evidence of a photonic 0.5th order fractional differentiator measuring the instantaneous frequency of a light pulse in the ten of ps regime. Next, we compare this measurement with another technique able to retrieve the instantaneous frequency profile [17]. To the best of our knowledge, this is the first work that experimentally demonstrates the convenience of using a fractional order differentiator for a specific task in the photonic domain, i.e. the instantaneous frequency measurement. This works opens the door for the use of fractional calculus operators solving specific problems in the photonic signal processing.

2. Theory

Let us suppose a given optical pulse, whose complex temporal envelope is given by $g(t)=|g(t)|\exp[j\varphi(t)]$, with $j=\sqrt{-1}$. Now, if we perform on this pulse not a standard, but a spectrally shifted 0.5th order fractional differentiation (with angular frequency shifting given by ω_s), the signal processed by the photonic fractional order differentiator could be written as $g(t)f(t)$, with $f(t)=\exp(j\omega_s t)$. Next, let us use the generalized Leibnitz rule for the differentiation of the product of two arbitrary functions:

$$\frac{d^r fg}{dt^r}=\sum_{q=0}^{\infty}\frac{\Gamma(r+1)}{\Gamma(r-q+1)q!}\frac{d^{r-q}f}{dt^{r-q}}\frac{d^q g}{dt^q}, \qquad (1)$$

where $\Gamma(.)$ is the gamma function.

Now, by replacing $r=1/2$, and taking into account that $d^a[\exp(j\omega_s t)]/dt^a=(j\omega_s)^a\exp(j\omega_s t)$, with $a\in\mathbf{R}$; it can be demonstrated that the instantaneous angular frequency profile of the input pulse is related to the intensities of the original plus the spectrally shifted 0.5th order differentiation $d^{0.5}/dt^{0.5}$ by the following approximation:

$$\frac{d\varphi(t)}{dt}\approx\frac{\left|\frac{d^{0.5}}{dt^{0.5}}[g(t)\exp(j\omega_s t)]\right|^2}{|g(t)|^2}-\omega_s, \qquad (2)$$

The mathematical details of this derivation can be followed in Appendix A of Ref. [19]. Eq. (2) shows that the instantaneous frequency profile can be obtained by simply dividing the temporal intensity profiles of the light pulse under test $|g(t)|^2$, and that of its corresponding spectrally-shifted 0.5th order fractional differentiation $\left|d^{0.5}[g(t)\exp(j\omega_s t)]/dt^{0.5}\right|^2$. The spectral shift ω_s should be high enough that the spectral content of the input pulse is mainly located at one side of the 0.5th order fractional differentiator resonance frequency. If required, the pulse's temporal phase profile can be obtained by numerical integration of Eq. (2), except by an undetermined numerical constant. It is worth noting the non-iterative nature of the proposed procedure; as opposed to other well-known techniques such as the Gerchberg-Saxton algorithm, which precludes real-time applications. On the contrary, the technique proposed here is potentially well-suited for real-time applications and non-repetitive events.

We deliberative postponed until now the characteristics required to a photonic fractional order differentiator, whose basic operation principle will be explained in the following. To this end, it is very useful to remind one property of the Fourier transform, namely:

$$\Im[g(t)]=G(\omega)\Rightarrow\Im\left[\frac{d^n g(t)}{dt^n}\right]=(j\omega)^n G(\omega), \qquad (3)$$

i.e. the Fourier transform of the nth time derivative of a given function is $(j\omega)^n$ times the original Fourier transform; where the $\Im$ symbol stands for the Fourier transform, and n is the order of differentiation, which is not necessarily restricted to be an integer. Therefore, and from a strictly spectral point of view, a 0.5th order fractional differentiator is essentially a high-pass filtering device with a transfer function given by $(j\omega)^{0.5}$ [3], where ω is the baseband angular frequency i.e. the difference between the optical angular frequency ω_{opt} and the central optical angular frequency of the signal ω_0.

3. Experimental

The photonic fractional order differentiation was performed in this work by using a long period fiber grating (LPG). A detailed characterization of this device working as a fractional order differentiator is out of the scope of this work; the interested reader can follow the fabrication details and performance characterization through Ref. [5]. Only for completeness, its main features will be summarized in the following. The LPG was inscribed in a boron doped photosensitive fiber (PS980 by Fibercore, numerical aperture of 0.13 and a cut-off wavelength of 980 nm) by using the point-by-point technique. The selected periodicity was of 187.6 µm, with a final LPG length of 146.5 mm. This LPG was specially fabricated to behave as a 0.5th order fractional differentiator around the resonance wavelength $\lambda_0=1035.5$ nm, with a -14 dB transmittance dip and a 3 dB bandwidth of 1.14 nm. The experimental measurement of the LPG transmission can be observed in Fig. 1 (in amplitude). In the same figure, it is compared with the theoretical amplitude response of an ideal 0.5th order fractional differentiator, i.e.. There is a good degree of resemblance between both within the whole operative optical bandwidth (determined by the first transmission maxima at both sides of the resonance dip), except at the resonance frequency, where the transmission should decay to $-\infty$. However, it should be emphasized that a slight deviation in the magnitude has lower consequences than a deviation of the phase

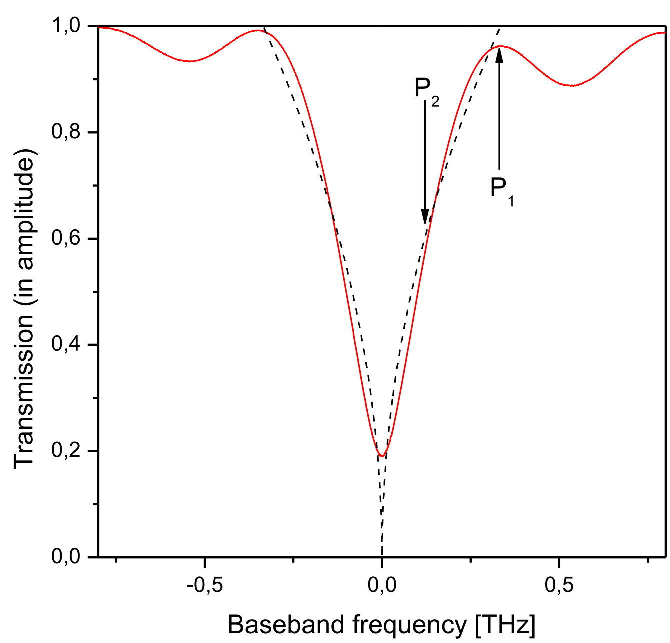

Fig. 1. Measured optical spectrum of the LPG (solid curve) and theoretical response (dashed curve) of a 0.5th order fractional differentiator, both in amplitude.