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Structure of polarimetric purity of a Mueller matrix and sources of depolarization

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ABSTRACT

The depolarization properties of a medium with associated Mueller matrix M are characterized through two complementary sets of parameters, namely 1) the three indices of polarimetric purity (IPP), which are directly linked to the relative weights of the spectral components of M and provide complete information on the structure of polarimetric randomness, but are insensitive to the specific polarimetric behaviors that introduce the lack of randomness, and 2) the set of three components of purity (CP), constituted by the polarizance, the diattenuation and the degree of spherical purity. The relations between these sets of physical invariant quantities are studied by means of their representation into a common purity figure. Furthermore, the polarimetric properties of a general Mueller matrix M are parameterized in terms of sixteen meaningful quantities, three of them being the IPP, which together with the CP provide complete information on the integral depolarization properties of the medium.

1. Introduction

Mueller polarimetry is applied for the analysis and study of a great variety of material samples in a continuously increasing number of areas of science, engineering, industry, medicine, etc. Nevertheless, the interpretation of the measured Mueller matrices, as well as the extraction of physical parameters containing decoupled information on the nature and properties of the sample is not a straightforward task. The polarimetric features of a material medium combine, in a complicated manner, polarizing, diattenuating, retarding and depolarizing effects. Therefore, the optimum knowledge of the structure of Mueller matrices is strongly required for the exploitation of polarimetric measurements.

This work is devoted to the the study of the depolarization properties of a material sample and describes how the two alternative approaches called the *indices of polarimetric purity* (hereafter IPP) [1] and the *components of purity* (hereafter CP) [2] are mutually related and can be jointly analyzed by means of their graphic representation into an common *purity figure*. For the sake of self-consistency and readability, this article is organized into the following sections. The present Section is devoted to the introduction of the main general notions involved in the further developments as well as the necessary conventions, terminology and notation. Sections 2 and 3 deal respectively with the definition and interpretation of the IPP and the CP of a medium with a given associated Mueller matrix $\mathbf{M}$. There are some polarimetric properties of a given material medium (including both IPP and CP) that remain invariant when the medium is serially combined with retarders, so that all so-called invariant-equivalent Mueller matrices, which constitute the subject of Section 4, share the same location in the purity figure. The purity figure, where the different types of Mueller matrices are represented according to the values of their CP and their IPP is studied in Section 5. Section 6 is dedicated to the analysis of the purity figures for the type-I and type-II *canonical depolarizers* [3]. These kinds of matrices are of special interest because they are representative of some intrinsic depolarizing properties of a given Mueller matrix $\mathbf{M}$. Section 7 deals with a parameterization of $\mathbf{M}$ in terms of meaningful phenomenological parameters that highlights the fact that depolarization is fully characterized, in quantity and quality, by means of five parameters, namely the three IPP, the diattenuation and the polarizance. Finally, Section 8 summarizes and discusses the main results and conclusions.

The interaction of a fully polarized beam with a medium that behaves as linear, deterministic, homogeneous and non-depolarizing, can be represented by means of the transformation $\mathbf{\Phi}' = \mathbf{T}\,\mathbf{\Phi}\,\mathbf{T}^{\dagger}$, where $\mathbf{\Phi}$, $\mathbf{\Phi}'$ are the input and output polarization matrices respectively, $\mathbf{T}$ is the Jones matrix that characterizes the polarimetric properties of the nondepolarizing medium for the given interaction conditions, and the dagger indicates conjugate transposed. This basic polarimetric interaction can also be expressed as $\mathbf{s}' = \mathbf{M}_J\,\mathbf{s}$ in terms of the corresponding input and output Stokes vectors ($\mathbf{s}$ and $\mathbf{s}'$ respectively) and the Mueller-Jones matrix (or *pure Mueller matrix*) $\mathbf{M}_J\,(\mathbf{T})$.

The physical polarimetric quantities that characterize this kind

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of *pure* systems can be easily identified by means of product (serial) decompositions of $\mathbf{T}$ (or $\mathbf{M}_J$), as for instance the polar decomposition [4] or the general serial decomposition [5,6]. Thus, the diattenuation, the polarizance and the retardance exhibited by the pure medium are easily decoupled and interpreted.

In general, the polarimetric behavior of a medium can be considered as a sort of incoherent convex sum, or ensemble average, of nondepolarizing interactions, in such a manner that a physical Mueller matrix can be expressed as a convex sum of pure Mueller matrices [7,8].

Leaving aside passivity constraints (i.e. the fact that naturally occurring phenomena do not increase the intensity of the incoming electromagnetic beams), two alternative, but equivalent, ways for the general characterization of Mueller matrices have been reported 1) the nonnegativity of the Hermitian matrix $\mathbf{H}$ (covariance or coherency matrix) associated with a given physical Mueller matrix (Cloude's criterion) [9,10], and 2) the nonnegativity of the N-matrix $\mathbf{G}\mathbf{M}^T\mathbf{G}\mathbf{M}$, where $\mathbf{G} \equiv \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric [11–15].

Thus, the structure of a general (depolarizing) Mueller matrix is rather more complicated than that of a pure Mueller matrix. In fact, unlike the transmittance (or reflectance) for unpolarized input light (hereafter called *mean intensity coefficient*), and unlike the diattenuation and polarizance properties, which are easily identified and defined from the given Mueller matrix $\mathbf{M}$ [16,17], the identification and parameterization of the depolarization properties are not so straightforward and require particular study and analysis.

Let us first recall that any Mueller matrix $\mathbf{M}$ can be expressed as [18]

$$\mathbf{M} = m_{00}\begin{pmatrix} 1 & \mathbf{D}^T \\ \mathbf{P} & \mathbf{m} \end{pmatrix}, \mathbf{D} \equiv \frac{1}{m_{00}}(m_{01}, m_{02}, m_{03})^T,$$
$$\mathbf{P} \equiv \frac{1}{m_{00}}(m_{10}, m_{20}, m_{30})^T, \mathbf{m} \equiv \frac{1}{m_{00}}\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}, \quad (1)$$

where $\mathbf{D}$ and $\mathbf{P}$ are the respective *diattenuation vector* and *polarizance vector* of $\mathbf{M}$. The absolute values of these vectors are the *diattenuation* $D \equiv |\mathbf{D}|$ and the *polarizance* $P \equiv |\mathbf{P}|$. Both polarizance P and diattenuation D have dual nature depending on the direction of propagation of light (forward or reverse) [10,19]; in fact, D is both the diattenuation of $\mathbf{M}$ and the polarizance of the *reverse Mueller matrix* $\mathbf{M}^r \equiv \mathrm{diag}(1, 1, -1, 1)\,\mathbf{M}^T\mathrm{diag}(1, 1, -1, 1)$ [20,21] ($\mathbf{M}^T$ being the transposed matrix of $\mathbf{M}$) corresponding to the same interaction as $\mathbf{M}$ but interchanging the input and output directions. The mean intensity coefficient of $\mathbf{M}$ is given by m_{00}. For some purposes, it is useful to use the following normalized version of $\mathbf{M}$

$$\hat{\mathbf{M}} \equiv \mathbf{M}/m_{00} = \begin{pmatrix} 1 & \mathbf{D}^T \\ \mathbf{P} & \mathbf{m} \end{pmatrix} \quad (2)$$

2. Components of purity

As a necessary step before further analyses, let us consider the notion of polarimetric purity. A given Mueller matrix $\mathbf{M}$ inherits, in some way, the statistical nature of the medium to which $\mathbf{M}$ is associated under given interaction conditions. Recall that a medium has a different associated Mueller matrix depending on 1) the spectral profile of the probe light beam; 2) the kind of interaction considered: refraction, reflection, scattering…; 3) the relative orientation of the medium with respect to the input beam; 4) the observation angle, etc. A medium that does not depolarize any totally polarized input beam is polarimetrically indistinguishable from a deterministic medium with well-defined Jones matrix $\mathbf{T}$ [and hence with well-defined *pure Mueller matrix* $\mathbf{M}_J(\mathbf{T})$] [5,22]. This kind of media is called *pure* or *nondepolarizing*. The closer is the polarimetric behavior to that of a nondepolarizing medium, the higher is its polarimetric purity.

A global measure of the *degree of polarimetric purity* of a medium is given by the *depolarization index* of $\mathbf{M}$ [16] defined as

$$P_\Delta = \sqrt{\left(D^2 + P^2 + 3P_S^2\right)/3} \quad (3)$$

in terms of the *components of purity* (CP), namely the *polarizance* P, the *diattenuation* D and the *degree of spherical purity* $P_S \equiv \|\mathbf{m}\|_2/\sqrt{3}$ [2] ($\|\mathbf{m}\|_2$ representing the Euclidean norm of the 3×3 submatrix $\mathbf{m}$). Conversely, an overall measure of the depolarizing power of a medium is given by the *depolarizance*

$$D_\Delta \equiv \sqrt{1 - P_\Delta^2} \quad (4)$$

Note that depolarizance was defined previously as $D_\Delta = 1 - P_\Delta$, but for reasons that are explained in Ref. [6], we consider more appropriate the indicated form.

Pure Mueller matrices are characterized by $P_\Delta = 1$ $(D_\Delta = 0)$, while Mueller matrices satisfying $P_\Delta < 1\,(D_\Delta > 0)$ are called *nonpure* or *depolarizing* Mueller matrices. A medium satisfying $P_\Delta = 0$ $(D_\Delta = 1)$ converts any input polarization state into a fully depolarized output one. Despite the fact that P_Δ is an objective overall measure of the polarimetric purity (lack of randomness of the polarimetric properties of the interaction represented by $\mathbf{M}$) it does not provide enough information for a complete parameterization of the polarimetric purity of $\mathbf{M}$.

While P and D measure the relative portions of purity due to polarizance and diattenuation properties respectively, P_S is a measure of the portion of purity that is not due to polarizance or diattenuation [2]. That is, the closer is $\hat{\mathbf{M}}$ to the Mueller matrix of a pure retarder (i.e., to an orthogonal Mueller matrix) the higher is the value of P_S. In fact, $P_S = 1$ if and only if $\hat{\mathbf{M}}$ is an orthogonal matrix. The value of P_S is restricted to $0 \le P_S \le 1$, where the lower limit $P_S = 0$ is reached when the submatrix $\mathbf{m}$ is just the zero matrix and therefore the corresponding Mueller matrix has the form of an *absolute partial polarizer-analyzer*

$$\mathbf{M} = m_{00}\begin{pmatrix} 1 & \mathbf{D}^T \\ \mathbf{P} & \mathbf{0} \end{pmatrix} \quad (5)$$

which is necessarily depolarizing since the minimum value of P_S compatible with total purity of $\mathbf{M}$ is $P_S = 1/\sqrt{3}$ [2]. A detailed study of the achievable values for P, D and P_S can be found in Ref [2].

Because of the common nature of P and D, and regardless the fact that they have respective specific and well-defined physical meanings, for some purposes it is useful to group them into the *degree of polarizance* [2]

$$P_P \equiv \sqrt{P^2 + D^2}/\sqrt{2} \quad (6)$$

which is an overall measure of the polarizing power of the system represented by the Mueller matrix $\mathbf{M}$ (both forward and reverse incidence directions being considered). The value of P_P is restricted to $0 \le P_P \le 1$, so that $P_P = 1$ corresponds to a total polarizer (the output states of both $\mathbf{M}$ and $\mathbf{M}^r$ are fully polarized regardless the degree of polarization of the input states). It should also be noted that pure diattenuators satisfy the condition $P_S^2 = 1 - 2P_P^2/3$ [2], so that a certain amount of spherical purity is consubstantial to this kind of systems. The value $P_P = 0$ is reached when the corresponding Mueller matrix $\mathbf{M}$ has zero polarizance and zero diattenuation.

Eq. (3) shows that the value of P_Δ is composed of the three complementary contributions of the corresponding *components of purity* P, D and P_S. Let us call *sources of purity* the quantities P_P and