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Focusing properties of phase-only generalized Fibonacci photon sieves

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ABSTRACT

We propose a new algorithm to extend the standard Fibonacci photon sieve to the phase-only generalized Fibonacci photon sieve (GFiPS) and find that the focusing properties of the phase-only GFiPS are only relevant to the characteristic roots of the recursion relation of the generalized Fibonacci sequences. By switching the transparent and opaque zones on the basis of the generalized Fibonacci sequences, we not only realize adjustable bifocal lengths, but also give their corresponding analytic expressions. Besides, we investigate a special phase-only GFiPS, a spiral-phase GFiPS, which can present twin vortices along the axial coordinate. Compared with the single focusing system, bifocal system can be exploited to enhance the processing speed, and offer a broad range of applications, such as direct laser writing, optical tweezers or atom trapping and paralleled fluorescence microscope.

1. Introduction

Focusing of X-ray and extreme ultraviolet (EUV) has many applications in physical and life sciences, such as high-resolution microscopy, spectroscopy, and lithography [1]. A traditional Fresnel zone plate (FZP), which has inherent limitations [2,3], can be used for this kind of focusing [4,5]. Some aperiodic zone plates, generated with the fractal Cantor set, have been proposed to overcome some of these limitations [6,7], another interesting mathematical generator of aperiodic zone plates is the Fibonacci sequence. Photonics is a potential field of applications for novel devices designed and constructed by using a Fibonacci sequence as a consequence of its unique properties. The focusing and imaging properties of Fibonacci optical elements, *e.g.*, quasicrystals [8,9], gratings [10–12], lenses [13–15], zone plates [16], *etc.*, are studied in detail. In mathematics, many mathematicians have extensively studied the Fibonacci sequence and its various generalizations [17,18] in the past decades.

In 2001, Kipp et al. proposed a photon sieve [19], which is a FZP with the transparent zones replaced by a great number of completely separated pinholes to overcome the disadvantages of traditional zone plate. Several kinds of theoretical models [20–22] have been studied mathematically and experimentally [23–26] to design different kinds of photon sieves, such as fractal [27,28], compound [23], Zernike apodized [29], phase zone [30], spiral [31], square [32], and reflection photon sieves [33]. In our previous work, we proposed a bifocal modified Fibonacci photon sieve (MFiPS), designed by using the Fibonacci sequence with two different initial seed elements [34], but the ratio of the two focal lengths is a fixed value.

In this paper, we introduce the aperiodic generalized Fibonacci sequences into photon sieves, and come to a conclusion that the phase-only generalized Fibonacci photon sieve (GFiPS) can generate two equal axial intensity foci with adjustable location. We find the relationship between the focusing properties of a phase-only GFiPS and its characteristic roots of the recursion relation of the generalized Fibonacci sequences. Besides, based on the laser vortex beams generated by use of spiral phase [13,35,36], we present a spiral-phase generalized Fibonacci photon sieve to produce twin vortices along the axial coordinate, which still has the same focusing properties as that mentioned above.

2. Phase-only generalized Fibonacci photon sieves

For the generalized Fibonacci sequences, their initial seed elements are as follows:

$$F_j = a_j \ \ (j = 1, 2, 3, a_j \in N^+). \tag{1}$$

And the corresponding linear recursion relation of the generalized Fibonacci sequences can be written as

$$F_n = pF_{n-1} + qF_{n-2} + rF_{n-3} \ \ (p, q, r \in R). \tag{2}$$

The absolute value of one of the corresponding characteristic roots of the recursion relation can be defined as the limit of the ratio of two consecutive generalized Fibonacci numbers

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$$\gamma = \lim_{j\to\infty} F_j/F_{j-1}. \quad (3)$$

When $(p, q, r)=(1, 1, 0)$, we can get the standard Fibonacci sequence. Obviously, $x_1=(1+\sqrt{5})/2$ and $x_2=(1-\sqrt{5})/2$, which are associated with the classical geometrical problem of the golden section, are its characteristic roots of the characteristic equation $x^2-x-1=0$.

We now retrospect the design of a FZP based on the plane wave incidence. All the designed rays are converged upon a single point. As known, the radius of the mth zone can be determined by [4]

$$r_m = \sqrt{(m\lambda/2)^2 + m\lambda f}, \quad (4)$$

where f denotes the expected focal length and λ is the incident wavelength.

A phase-only GFiPS can be generated similar to the process of a traditional photon sieve (TPS) [23] and a modified Fibonacci photon sieve [34]. Taking a generalized Fibonacci sequence into account, whose initial seed elements are $F_1=1$, $F_2=2$ and $F_3=3$ and the recursion relation is $F_n=-F_{n-1}+F_{n-2}-F_{n-3}$, where the minus denotes complement operation. After encoding two seed elements as $(F^1, F^2, F^3)=(0, 01, 101)$, the six-order switching sequence F^4 is 010011 while 1 denotes transparent zones and 0 denotes opaque ones. That means the number of total zones is six and three zones are transparent. The corresponding phase-only GFiPS is shown in Fig. 1(b), which is a generalized Fibonacci zone plate (GFiZP) [see Fig. 1(a)] with the transparent and opaque zones replaced by a great number of completely separated pinholes, and their phases are 0 and π, respectively. The diameter d of the pinholes in the mth zone of w in width should take the value of $d=1.16w$ [34].

3. Simulation and discussion

Theoretically, the diffraction field of the phase-only GFiPS can be numerically calculated by the Rayleigh–Sommerfeld diffraction integral formula under the condition of a plane wave incidence with unit amplitude [37–39]

$$U(x, y, z) = \iint_{\Sigma} t(\xi, \eta, 0)\frac{\exp(ikR)}{i\lambda R}\left(1 + \frac{i}{kR}\right)\frac{z}{R}\mathrm{d}\xi\mathrm{d}\eta, \quad (5)$$

where $t(\xi, \eta, 0)$ is the pupil function of a GFiPS, i is the imaginary unit, k is the wave number, z is the axial distance from the pupil plane, and R denotes the distance between point $(\xi, \eta, 0)$ and point (x, y, z).

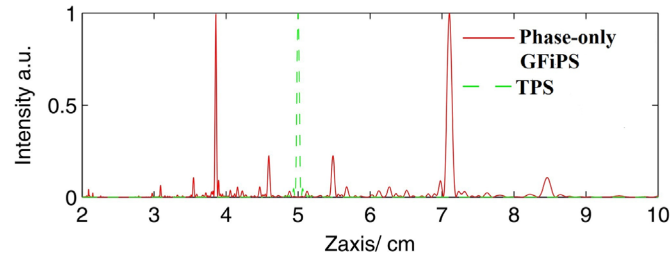

Fig. 2. Normalized intensity distribution along the optical axis produced by a phase-only GFiPS and a TPS with the same resolution.

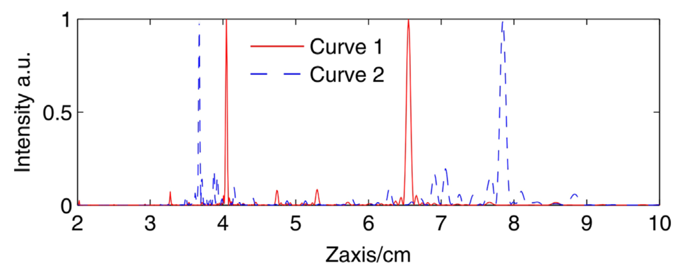

Fig. 3. Normalized axial intensity distribution produced by two phase-only GFiPSs based on different generalized Fibonacci sequences. Curve 1: $F_n=-F_{n-1}+F_{n-2}$ (F^{11} in this case); curve 2: $F_n=2F_{n-1}+0.3F_{n-2}$ (F^8 in this case).

Table 1
The focusing properties of phase-only GFiPSs.

Sequences		(p, q, r)		
		(−1,1,−1)	(−1,1,0)	(2,0.3,0)
Total zones		230	233	241
γ		|−1.839|	|−1.618|	2.140
a (mm)		2.716	2.716	2.762
Focal lengths based on Eq. (6)	**f_I (cm)**	3.862	4.048	3.670
	f_{II} (cm)	7.103	6.549	7.854
Focal lengths based on numerical calculation	**f_1 (cm)**	3.861	4.047	3.675
	f_2 (cm)	7.107	6.549	7.849
	f_2/f_1	1.841	1.618	2.136

To investigate the focusing performance of the phase-only GFiPSs, some numerical simulations are done with the incident wavelength λ of 632.8 nm, the expected focal length f of 5 cm. Based on Eq. (5), we use FFT method in the following simulation in HP Z800 workstation.

First of all, a phase-only GFiPS of ten-order switching sequence F^{10} is discussed. Its encoded seed elements are $(F^1, F^2, F^3)=(0, 01, 011)$,

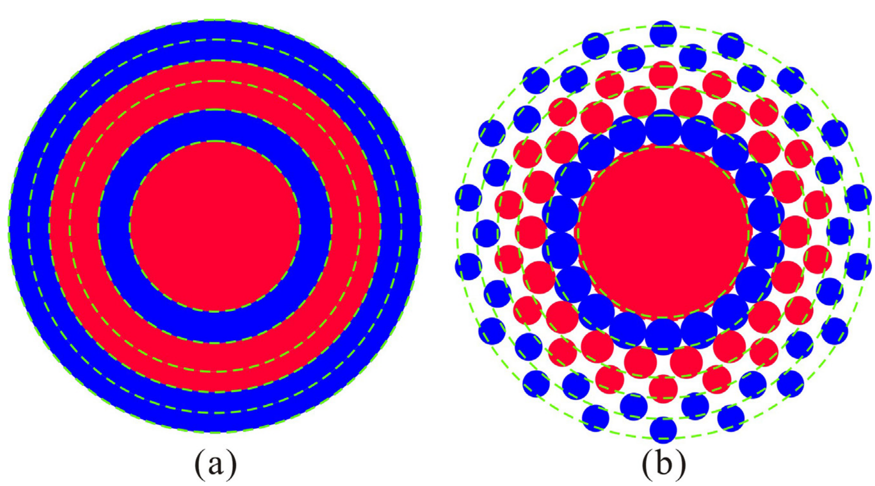

Fig. 1. (a) A generalized Fibonacci zone plate. (b) A phase-only generalized Fibonacci photon sieve.