



From wrinkling to global buckling of a ring on a curved substrate



R. Lagrange^a, F. López Jiménez^b, D. Terwagne^{b,1}, M. Brojan^{c,2}, P.M. Reis^{b,c,*}

^a Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

^b Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

^c Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

ARTICLE INFO

Article history:

Received 21 August 2014

Received in revised form

12 December 2015

Accepted 7 February 2016

Available online 18 February 2016

Keywords:

Elasticity

Instability

Buckling

Wrinkling

Ring

Substrate

ABSTRACT

We present a combined analytical approach and numerical study on the stability of a ring bound to an annular elastic substrate, which contains a circular cavity. The system is loaded by depressurizing the inner cavity. The ring is modeled as an Euler–Bernoulli beam and its equilibrium equations are derived from the mechanical energy which takes into account both stretching and bending contributions. The curvature of the substrate is considered explicitly to model the work done by its reaction force on the ring. We distinguish two different instabilities: periodic wrinkling of the ring or global buckling of the structure. Our model provides an expression for the critical pressure, as well as a phase diagram that rationalizes the transition between instability modes. Towards assessing the role of curvature, we compare our results for the critical stress and the wrinkling wavelength to their planar counterparts. We show that the critical stress is insensitive to the curvature of the substrate, while the wavelength is only affected due to the permissible discrete values of the azimuthal wavenumber imposed by the geometry of the problem. Throughout, we contrast our analytical predictions against finite element simulations.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

Wrinkling is a stress-driven mechanical instability that occurs when a stiff and slender surface layer, bonded to a compliant substrate, is subject to compression. This universal instability phenomenon is found in numerous natural and technological/engineering examples, over a wide range of length scales, including carbon nanotubes (Lourie et al., 1998), pre-stretched elastomers used in flexible electronics applications (Kim et al., 2011), human skin (Chen and Yin, 2010), drying fruit (Yin et al., 2009), surface morphology of the brain (Budday et al., 2014) and mountain topographies generated due to tectonic stresses (Price and Cosgrove, 1990; Huddleston and Lan, 1993).

Over the past decade, there has been an upsurge of interest in the study of the mechanics of wrinkling, along with a change of paradigm in regarding surface instabilities as an opportunity for functionality, instead of a first step in the route to structural failure (Genzer and Groenewold, 2006; Li et al., 2012). The first mechanical studies of wrinkling were motivated

* Corresponding author at: Departments of Mechanical Engineering and Civil & Environmental Engineering, Massachusetts Institute of Technology, #1-330, 77 Massachusetts Avenue, Cambridge, MA 02139, USA

E-mail address: preis@mit.edu (P.M. Reis).

¹ Current address: Department of Physics, Faculté des Sciences, Université Libre de Bruxelles (ULB), Bruxelles 1050, Belgium.

² Current address: Faculty of Mechanical Engineering, University of Ljubljana, Slovenia.

by the stability of sandwich panels (Allen, 1969), used in lightweight structural applications, in which the core acts as a soft substrate for the much stiffer skin. More recently, Bowden et al. (1998) showed how the wrinkling of a thin film on an elastomeric substrate can be used to produce complex self-organized patterns. Their seminal work has instigated the realization of wrinkling through several different actuation mechanisms, including thermal mismatch (Huck et al., 2000), tissue growth/atrophy (Ben Amar and Goriely, 2005; Li et al., 2011a; Budday et al., 2014), swelling by a liquid (Chan and Crosby, 2006b) or vapor solvent (B Reid and Crosby, 2009), and pneumatics (Terwagne et al., 2014). The opportunities in applications opened by such a wide range of external stimuli have enabled the usage of wrinkling in photonics (Kim et al., 2012), optics (Chan and Crosby, 2006a), self-assembly (Yoo et al., 2002), microfluidics (Yin et al., 2012) and morphogenesis (Efimenko et al., 2005).

In order to provide a theoretical background to these recent developments, several authors have built on the pioneering work of Allen (1969), who first provided closed form solutions for the critical stress and wavelength obtained when an initially straight beam, adhered to an infinite plane substrate, is placed under a state of uniaxial compression. Chen and Hutchinson (2004) extended this work to consider the case of a plate adhered to a flat substrate under equi-biaxial compression and performed a nonlinear analysis of the Föppl–von Kármán (Landau and Lifshitz, 1959; Timoshenko and Gere, 1961). Huang et al. (2005) further refined these efforts by considering the effect of a finite substrate. Both studies showed the existence of multiple buckling modes associated with the same value of critical stress. The stability of these modes under different loading conditions has been addressed by Audoly and Boudaoud (2008a,b,c), who produced a stability diagram covering the evolution from low to high values of overstress. However, experiments by Cai et al. (2011) found disagreement at low values of overstress, suggesting that a finite intrinsic curvature of their experimental system, even if small, may play an important role in dictating pattern selection.

Early studies of wrinkling on curved substrates, as in the flat configuration, were also motivated by a structural problem; in this case, in the context of the stability of the outer shell of rockets (Kachman, 1959; Seide and Weingarten, 1961; Seide, 1962). More recent studies that consider instabilities as a possible source of functionality have led to applications of curved configurations in adhesion (Kundu et al., 2011), microfluidics (Mei et al., 2010), morphogenesis of microparticles (Yin et al., 2014), optics (B Reid and Crosby, 2013) and aerodynamic drag reduction (Terwagne et al., 2014). Curvature also plays a relevant role in the growth of biological systems (Li et al., 2011b). Despite these important emerging applications, the mechanics of wrinkling on curved substrates remains poorly understood, when compared to the planar counterpart.

Systematic Finite Element simulations of wrinkling in curved systems have been performed (Yin et al., 2009; Chen and Yin, 2010; Li et al., 2011c; Cao et al., 2012) that highlighted a complex pattern formation process. These numerical studies also suggested the possibility for curvature to affect the selected patterns and modify the relevant characteristic length scales, which calls for a robust theoretical backing. Analytical predictions are challenged by the difficulty of modeling the stiffness of the substrate, even in two-dimensional configurations. Cheng (1996) and Cai et al. (2011) used the stiffness provided by Allen (1969) for the flat case, such that their model therefore neglects the contribution of curvature on the response of the substrate. Yin et al. (2009) used the prediction provided by Brush and Almroth (1975), which accounts for curvature but does not consider its influence on the wrinkling wavelength and their prediction does not converge to the classical planar case when the curvature tends to zero. As such, there is a need to quantify the effect of curvature on the stiffness of the substrate and its subsequent influence on wrinkling.

Here, to the best of our knowledge, we provide the first analytical work that accounts for both the curvature of a (2D) shell–substrate system, as well as the finite size of the substrate. As an initial step, we focus our study on a curved film adhered to a cylindrical substrate, instead of dealing with non-zero Gaussian curvature geometries, which is left for a future study. We assume axial-symmetry to further simplify the system to the 2D problem of a ring on an annular substrate. Mechanical loading is applied by depressurizing a circular cavity inside the substrate, which places the system under a state of compression. This geometry is motivated by recent experiments (Terwagne et al., 2014) that demonstrated the usage of wrinkling on spherical samples for switchable and tunable aerodynamic drag reduction. In our simplified 2D system, we solve the elasticity problem for the substrate and derive a closed form expression for its stiffness, which is then used in the stability analysis of the ring to quantify the buckling patterns.

The paper is organized as follows: In Section 2, we introduce our system along with its material and geometrical parameters. We also describe the possible instability modes, and present a simplified phase diagram, with the aim of providing physical insight on the problem. In Section 3, we then introduce the kinematics of the ring attached to the substrate and determine the stiffness of the substrate. We proceed by defining a strain energy that includes both bending and stretching of the ring, as well as the effect of the substrate. Energy minimization yields the equilibrium equations of the problem. An asymptotic expansion is then used to calculate the principal solution and the bifurcation at the onset of instability. In Section 4, we describe the finite element simulations that we have performed for this same system.

The results of our investigation are presented in Section 5. Throughout, we directly compare the analytical predictions to the numerical simulations. We start with the fundamental solution and the critical conditions that lead to instability. We then construct a phase diagram which rationalizes the dependence of the instability modes on the governing parameters. The results for our system are then quantitatively compared to those for wrinkling of a film on a planar substrate, highlighting the effect of curvature. Finally, Section 6 summarizes our findings and provides perspectives for potential extensions of our work in future studies.

Download English Version:

<https://daneshyari.com/en/article/792844>

Download Persian Version:

<https://daneshyari.com/article/792844>

[Daneshyari.com](https://daneshyari.com)