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Dynamic instabilities of frictional sliding at a bimaterial interface

Efim A. Brener [a], Marc Weikamp [b], Robert Spatschek [b,c], Yohai Bar-Sinai [d], Eran Bouchbinder [d,*]

[a] Peter Grünberg Institut, Forschungszentrum Jülich, D-52425 Jülich, Germany
[b] Max-Planck-Institut für Eisenforschung GmbH, D-40237 Düsseldorf, Germany
[c] Institute for Energy and Climate Research, Forschungszentrum Jülich GmbH, D-52425 Jülich, Germany
[d] Chemical Physics Department, Weizmann Institute of Science, Rehovot 7610001, Israel

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ABSTRACT

Understanding the dynamic stability of bodies in frictional contact steadily sliding one over the other is of basic interest in various disciplines such as physics, solid mechanics, materials science and geophysics. Here we report on a two-dimensional linear stability analysis of a deformable solid of a finite height H, steadily sliding on top of a rigid solid within a generic rate-and-state friction type constitutive framework, fully accounting for elastodynamic effects. We derive the linear stability spectrum, quantifying the interplay between stabilization related to the frictional constitutive law and destabilization related both to the elastodynamic bi-material coupling between normal stress variations and interfacial slip, and to finite size effects. The stabilizing effects related to the frictional constitutive law include velocity-strengthening friction (i.e. an increase in frictional resistance with increasing slip velocity, both instantaneous and under steady-state conditions) and a regularized response to normal stress variations. We first consider the small wave-number k limit and demonstrate that homogeneous sliding in this case is universally unstable, independent of the details of the friction law. This universal instability is mediated by propagating waveguide-like modes, whose fastest growing mode is characterized by a wave-number satisfying $kH \sim O(1)$ and by a growth rate that scales with H^{-1}. We then consider the limit $kH \to \infty$ and derive the stability phase diagram in this case. We show that the dominant instability mode travels at nearly the dilatational wave-speed in the opposite direction to the sliding direction. In a certain parameter range this instability is manifested through unstable modes at all wave-numbers, yet the frictional response is shown to be mathematically well-posed. Instability modes which travel at nearly the shear wave-speed in the sliding direction also exist in some range of physical parameters. Previous results obtained in the quasi-static regime appear relevant only within a narrow region of the parameter space. Finally, we show that a finite-time regularized response to normal stress variations, within the framework of generalized rate-and-state friction models, tends to promote stability. The relevance of our results to the rupture of bi-material interfaces is briefly discussed.

* Corresponding author.
E-mail address: Eran.Bouchbinder@weizmann.ac.il (E. Bouchbinder).

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1. Background and motivation

The dynamic stability of steady-state homogeneous sliding between two macroscopic bodies in frictional contact is a basic problem of interest in various scientific disciplines such as physics, solid mechanics, materials science and geophysics. The emergence of instabilities may give rise to rich dynamics and play a dominant role in a broad range of frictional phenomena (Ruina, 1983; Ben-Zion, 2001; Scholz, 2002; Ben-Zion, 2008; Gerde and Marder, 2001; Ibrahim, 1994a,b;Di Bartolomeo et al., 2010; Tonazzi et al., 2013; Baillet et al., 2005; Behrendt et al., 2011; Meziane et al., 2007). The response of a frictional system to spatiotemporal perturbations, and the accompanying instabilities, are governed by several physical properties and processes. Generally speaking, one can roughly distinguish between bulk effects (i.e. the constitutive behavior and properties of the bodies of interest, their geometry and the external loadings applied to them) and interfacial effects related to the frictional constitutive behavior. The ultimate goal of a theory in this respect is to identify the relevant physical processes at play, to quantify the interplay between them through properly defined dimensionless parameters and to derive the stability phase diagram in terms of these parameters, together with the growth rate of various unstable modes.

As a background and motivation for what will follow, we would like first to briefly discuss the various players affecting the stability of frictional sliding, along with stating some relevant results available in the literature. Focusing first on bulk effects, it has been recognized that when considering isotropic linear elastic bodies, there is a qualitative difference between sliding along interfaces separating bodies made of identical materials and interfaces separating dissimilar materials. In the former case, there is no coupling between interfacial slip and normal stress variations, while in the latter case — due to broken symmetry — such coupling exists (Comninou, 1977a,b; Comninou and Schmueser, 1979; Weertman, 1980; Andrews and Ben-Zion, 1997; Ben-Zion and Andrews, 1998; Adams, 2000; Cochard and Rice, 2000; Rice et al., 2001; Ranjith and Rice, 2001; Gerde and Marder, 2001; Adda-Bedia and Ben Amar, 2003; Ampuero and Ben-Zion, 2008). This coupling may lead to a reduction in the normal stress at the interface and consequently to a reduction in the frictional resistance. Hence, bulk material contrast (i.e. the existence of a bi-material interface) potentially plays an important destabilizing role in the stability of frictional sliding. Another class of bulk effects is related to the finite geometry of any realistic sliding bodies and the type of loading applied to them (e.g. velocity or stress boundary conditions). To the best of our knowledge, the latter effects are significantly less explored in the literature (but see Rice and Ruina, 1983; Ranjith, 2009, 2014).

In relation to interfacial effects, it has been established that sliding along a bi-material interface described by the classical Coulomb friction law, $\tau = \sigma f$ (τ is the local friction stress, σ is the local compressive normal stress and f is a constant friction coefficient), is unstable against perturbations at all wavelengths and irrespective of the value of the friction coefficient, when the bi-material contrast is such that the generalized Rayleigh wave exists (Ranjith and Rice, 2001). The latter is an interfacial wave that propagates along frictionless bi-material interfaces, constrained not to feature opening (Weertman, 1963; Achenbach and Epstein, 1967; Adams, 1998; Ranjith and Rice, 2001). It is termed the generalized Rayleigh wave because it coincides with the ordinary Rayleigh wave when the materials are identical and it exists when the bi-material contrast is not too large. In fact, the response to perturbations in this case is mathematically ill-posed (Renardy, 1992; Adams, 1995; Martins and Simões, 1995; Martins et al., 1995; Simões and Martins, 1998; Ranjith and Rice, 2001). Ill-posedness, which is a stronger condition than instability (i.e. all perturbation modes can be unstable, yet a problem can be mathematically well-posed), will be discussed later. It has been then shown that replacing Coulomb friction by a friction law in which the friction stress τ does not respond instantaneously to variations in the normal stress σ, but rather approaches $\tau = \sigma f$ over a finite time scale, can regularize the problem, making it mathematically well-posed (Ranjith and Rice, 2001).

Subsequently, the problem has been addressed within the constitutive framework of rate-and-state friction models, where the friction stress depends both on the slip velocity and the structural state of the interface. Within this framework (Dieterich, 1978, 1979; Ruina, 1983; Rice and Ruina, 1983; Heslot et al., 1994; Marone, 1998; Berthoud et al., 1999; Baumberger and Berthoud, 1999; Baumberger and Caroli, 2006), the simplest version of the friction stress takes the form $\tau = \sigma f(\phi, v)$, where v is the difference between the local interfacial slip velocities of the two sliding bodies and ϕ is a dynamic coarse-grained state variable.[1] Under steady-state sliding conditions the state variable ϕ attains a steady-state value determined by v, $\phi_0(v)$. Within such a constitutive framework, the most relevant physical quantities for the question of stability, which will be extensively discussed below, are the instantaneous response to variations in the slip velocity, $\partial_v f(\phi, v)$ (the so-called "direct effect"), and the variation of the steady-state frictional strength with the slip velocity, $d_v f(\phi_0(v), v)$ (Rice et al., 2001). Note that here and below we use the following shorthand notation: $d_v \equiv d/dv$ and $\partial_v \equiv \partial/\partial v$.

Previous studies have argued that an instantaneous strengthening response, i.e. the experimentally well-established positive direct effect $\partial_v f(\phi, v) > 0$ (which is associated with thermally activated rheology (Baumberger and Caroli, 2006)), is sufficient to give rise to the existence of a quasi-static range of response to perturbations at sufficiently small slip velocities (Rice et al., 2001). The existence of such a quasi-static regime is non-trivial (e.g. it does not exist for Coulomb friction); it implies that when very small slip velocities are of interest, one can reliably address the stability problem in the framework of quasi-static elasticity (i.e. omitting inertial terms to begin with), rather than considering the full — and more difficult — elastodynamic problem and then take the quasi-static limit. Within such a quasi-static framework, it has been shown that $\partial_v f(\phi, v) > 0$ can lead to stable response against sufficiently short wavelength perturbations, even if the interface is velocity-weakening in steady-state, $d_v f(\phi_0(v), v) < 0$ (Rice et al., 2001). Furthermore, it has been shown that sufficiently strong

[1] In principle there can be more than one internal state variables, but we do not consider this possibility here.