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Finite size and geometrical non-linear effects during crack pinning by heterogeneities: An analytical and experimental study

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ABSTRACT

Crack pinning by heterogeneities is a central toughening mechanism in the failure of brittle materials. So far, most analytical explorations of the crack front deformation arising from spatial variations of fracture properties have been restricted to weak toughness contrasts using first order approximation and to defects of small dimensions with respect to the sample size. In this work, we investigate the non-linear effects arising from larger toughness contrasts by extending the approximation to the second order, while taking into account the finite sample thickness. Our calculations predict the evolution of a planar crack lying on the mid-plane of a plate as a function of material parameters and loading conditions, especially in the case of a single infinitely elongated obstacle. Peeling experiments are presented which validate the approach and evidence that the second order term broadens its range of validity in terms of toughness contrast values. The work highlights the non-linear response of the crack front to strong defects and the central role played by the thickness of the specimen on the pinning process.

1. Introduction

Predicting the role played by small scale heterogeneities on the macroscopic response of solids is an important challenge in mechanics. In a large range of free-boundary and free-discontinuity problems like wetting of liquids on solid substrates, magnetization of ferromagnetic materials or phase transformations, the relation between microscopic properties and macroscopic behavior is governed by some interface which can be highly sensitive to localized defects. In the context of brittle fracture, this reflects on the strong dependence of the toughness to microstructural details, like the spatial distribution of defects and their strength (Gao and Rice, 1989; Bower and Ortiz, 1991; Xia et al., 2012; Patinet et al., 2013b; Démery et al., 2014). The development of predictive tools connecting microstructural parameters of materials to their failure properties is central to engineer materials achieving increased resistance and lifetime. However, to address this challenge, predictive models must consider realistic situations and overcome the limitations of current theories that consider either weak variations of material properties or infinitely small defects compared to the specimen dimensions. The extension of

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these theories to strong heterogeneities and finite size specimens is the central point of this work.

Microstructural effects on brittle failure phenomena have been largely explored through the fracture mechanics analysis of a continuum elastic medium characterized by some heterogeneous field of local toughness. This approach has met a fair amount of success in capturing experimental observations (Ponson and Bonamy, 2010; Dalmas et al., 2010; Santucci et al., 2010; Xia et al., 2012; Patinet et al., 2013a). In this description, small scale variations of the failure properties locally perturb crack propagation and, *in fine*, affect the whole failure behavior of the material. The central point in this approach is the prediction of the crack front geometry from the characteristics of local toughness field (spatial distribution of defects, toughness contrast, *etc.*) and the distribution of the local stress intensity factor along a perturbed crack front (Lazarus, 2011).

Thirty years ago, Rice (1985) derived a first-order formula for the variations of the stress intensity factor induced by some small, but otherwise arbitrary coplanar perturbation of the front of a semi-infinite tensile crack in an infinite body. This expression has been used extensively to predict planar crack growth evolution through random arrays of defects (Schmittbuhl et al., 1995; Ramanathan et al., 1997; Bonamy et al., 2008), and decipher the puzzling geometrical properties of planar cracks observed in experiments (Delaplace et al., 1999; Santucci et al., 2010; Bonamy and Bouchaud, 2011).

However, Rice's first-order formula relies on the assumption that crack front perturbations are of small wavelength compared to the specimen dimensions, which is questionable in some experiments (Schmittbuhl et al., 2003). This was the motivation for Legrand et al.'s (2011) recent extension of Rice's (1985) formula to the case of coplanar perturbation of an emerging tensile crack lying on the mid-plane of a plate of finite thickness, thus accounting for the effect of the finite dimensions of the specimen. Patinet et al. (2013a) showed that the new formula did significantly improve the agreement between experimental and computed shapes of crack fronts deformed by the presence of obstacles.

Both Rice's (1985) and Legrand et al.'s (2011) formulae are however accurate only to first order in the perturbation of the front, which limits their application to weak variations of toughness. To explore the non-linear response of cracks pinned by defects of larger contrasts, Leblond et al. (2012) extended Rice's (1985) first-order formula for a semi-infinite crack in an infinite body to second order, under the assumption of independence of the unperturbed stress intensity factor imposed by the loading with respect to the average crack front location. Then, Vasoya et al. (2013) released this hypothesis, thus extending the range of application of Leblond et al.'s (2012) formula to general loading conditions. Recently, Willis (2013) and Willis and Movchan (2014) investigated the dynamic perturbation of a crack up to second order. Their results, taken in the elastostatic limit for mode I loading conditions, were found to be consistent with those previous works.

Still, since both Leblond et al.'s (2012) and Vasoya et al.'s (2013) works considered only infinite bodies, the combined effect of the finite size of the specimens and the geometrical nonlinearities induced by strong defects remains unexplored, and it is the aim of this work to address this question. The results of our calculations apply to several experimental situations involving strong heterogeneities, and for which the crack front perturbation wavelength compares with the thickness of the specimen (Santucci et al., 2010; Chopin et al., 2011). This study also provides predictive tools relevant for the design of heterogeneous materials or thin films where crack pinning by strong engineered obstacles is used to control and modify their failure properties, *e.g.* to produce asymmetry in the peeling strength of adhesives (Xia et al., 2012).

The paper is organized as follows:

- Section 2 recalls, as a prerequisite, some established results for planar cracks with slightly perturbed fronts. These results pertain to cracks located (i) in some arbitrary body (Rice, 1989) and (ii) on the mid-plane of a plate (Legrand et al., 2011).
- Section 3 presents an extension of Legrand et al.'s (2011) first-order results for a cracked plate to the second order, using Rice's (1989) general results.
- Section 4 applies these results to the case of a semi-infinite crack propagating quasi-statically along the mid-plane of a plate of finite thickness having some heterogeneous distribution of toughness. Assuming the stress intensity factor to be equal to the toughness at every point of the crack front, we determine the resulting shape of this front up to second order in the toughness fluctuations.

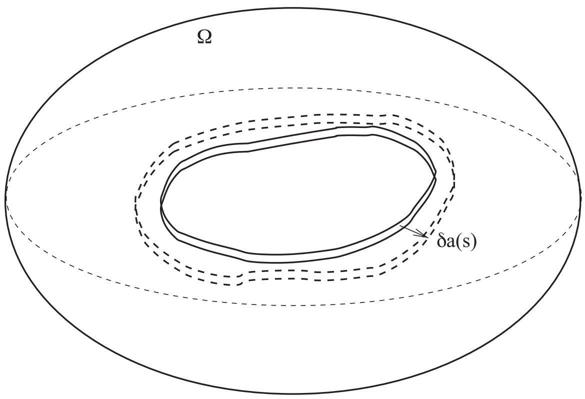

Fig. 1. A planar mode I crack with a slightly perturbed front in an arbitrary body.