



# Changes of skewness and sharpness of partially coherent decentered annular beams on propagation



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## ABSTRACT

Changes of skewness and sharpness of partially coherent decentered annular beams (PCDA beams) on propagation both in free space and in oceanic turbulence are studied. Based on the Wigner distribution function, the analytical expressions for the skewness parameter  $A$  and the kurtosis parameter  $K$  of PCDA beams are derived. The analytical expression for the oceanic turbulence parameter  $T$  related to  $K$  is also derived, and characteristics of  $T$  are examined. It is found that the behaviors of  $A$  and  $K$  in oceanic turbulence are quite different from those in free space. In free space, the mass of the intensity distribution may move from one side of the centroid position axis  $y_c$  to another side at a certain propagation distance  $z_0$ , and  $z_0$  is independent of the correlation parameter  $\tau$ . The mass of the intensity distribution is concentrated on one side of  $y_c$  on propagation only for a poorly coherent beam in free space, but it is always this situation for different value of  $\tau$  when oceanic turbulence is not weak. In free space, it takes a leptokurtic profile in the far field, and a Gaussian profile appears only for a poorly coherent beam. However, in oceanic turbulence it always reaches a Gaussian profile for different value of  $\tau$  in the far field.

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## 1. Introduction

It is known that the second-order-moments-based beam propagation factor ( $M^2$  factor) is a useful parameter characterizing different laser beams [1]. But it is insufficient, since no information concerning the shape features of a laser beam, such as the symmetry and degree of flatness, is inferred from the  $M^2$  factor. The knowledge of the symmetry and degree of flatness of laser beams can be obtained from the higher-order moments [2,3]. For example, the skewness parameter  $A$  related to third-order and second-order moments is used to describe the symmetry of a laser beam [4], and the kurtosis parameter  $K$  related to fourth-order and second-order moments is used to characterize the sharpness (or flatness) of a laser beam [5]. Generally, parameter  $A=0$  (i.e., symmetry profile) remains unchanged on propagation if a laser beam is symmetric and the media is isotropic; while the  $K$  parameter changes on propagation for all laser beams except for Gaussian, Hermit–Gaussian and Laguerre–Gaussian beams, their shape remains unchanged on propagation. Several works were carried out concerning the sharpness changes of laser beams propagating in free space and through ABCD optical systems [2,3,5–7]. But, vary a few of paper examined the skewness of a

light beam [8].

Annular beams with decentered field will be encountered in practice. For example, annular beams exist in many telescopes [9], and high-power lasers use unstable optical resonators as resonant cavities which produce an annular beam with decentered field [10]. It is important to study the propagation of laser beams through atmospheric turbulence for many practical applications [11]. We examined the centroid position related to first-order moments of partially coherent decentered annular beams (PCDA beams) propagating atmospheric turbulence [12]. Several works were performed concerning the characteristics related to second-order moments of coherent and partially coherent annular beams propagating atmospheric turbulence [13,14]. Recently, we study propagation characteristics of decentered annular beams propagating atmospheric turbulence [15], where only the fully coherent decentered annular beam is considered. In practice, particularly in high-power technology, a partially coherent laser source is often encountered. Furthermore, skewness and sharpness will change on propagation due to partially coherence of a laser beam.

Since recently the interest in active optical underwater communications, imaging and sensing appeared [16], it has become important to deeply understand how the oceanic turbulence affects light propagation properties. In 2000, Nikishov proposed the power spectrum of oceanic turbulence [17]. And then after that, several studies on light propagation through oceanic turbulence were carried out. For example, Lu et al studied the influence of

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temperature and salinity fluctuations on propagation behavior of partially coherent beams in oceanic turbulence [18], Korotkova et al examined the effect of oceanic turbulence on polarization and spectral changes of stochastic beams [19,20], Zhao et al investigated the propagation of radially polarized beams and stochastic electromagnetic vortex beams through oceanic turbulence [21,22], Baykal et al studied scintillations of optical plane and spherical waves and multimode laser beams in underwater turbulence [23,24], Zhang et al examined the evolution behavior of Gaussian Schell-model vortex beams propagating through oceanic turbulence [25], and we studied the influence of oceanic turbulence on propagation characteristics of Gaussian array beams [26]. However, the beam characteristics of third-order and fourth-order moments on propagation in oceanic turbulence haven't been examined until now. In this paper, changes of skewness and sharpness related to third-order and fourth-order moments of PCDA beams on propagation both in free space and in oceanic turbulence are studied in detail. Some interesting results are obtained, and some explanations are also given using the average intensity distribution.

## 2. Formulae of higher-order moments

The field of decentered annular beams at the source plane  $z=0$  can be written as [12,27]

$$E(\rho'_1, 0) = (1 - \beta x'_1) \left[ \sum_{t=1}^M \alpha_t \exp\left(-\frac{t\rho_1'^2}{w_0^2}\right) - \sum_{t=1}^M \alpha_t \exp\left(-\frac{t\rho_1'^2}{w_0'^2}\right) \right], \quad (1)$$

where  $\rho'_1 = (x'_1, y'_1)$  is the transverse vector at the source plane  $z=0$ ,  $w_0$  is the waist width of the Gaussian mode at the source plane  $z=0$ ,  $M$  is the beam order, and  $\alpha_t = (-1)^{t+1} M! / [t!(M-t)!]$ ,  $w_0' = \varepsilon w_0$  and  $0 < \varepsilon < 1$ ,  $\varepsilon$  is called the obscure ratio and  $\beta$  is called the decentered parameter.

The cross spectral density function of PCDA beams at the source plane  $z=0$  is written as

$$W(\rho'_1, \rho'_2, 0) = E(\rho'_1, 0) E^*(\rho'_2, 0) \mu(\rho'_1, \rho'_2, 0), \quad (2)$$

where  $\mu(\rho'_1, \rho'_2, 0) = \exp[-(\rho'_1 - \rho'_2)^2 / 2\sigma_0^2]$  is the spectral degree of coherence with Gaussian term, with  $\sigma_0$  being the spatial correlation length of the source.

When  $\rho'_1 = \rho'_2$ , Eq. (2) reduces to the expression for the intensity. The counter lines of the intensity at the source plane  $z=0$  for different values of  $\beta$  and  $\varepsilon$  are shown in Fig. 1(a)–(c), respectively. From Fig. 1 it can be seen that the larger  $\beta$  means the beam centroid position is further away from the center, the larger  $\varepsilon$  means the thinner beam is.

The Wigner distribution function (WDF) is very useful for handling partially coherent beams, which can be defined in terms

of the cross-spectral density as [28]

$$h(\rho, \theta, z) = \left(\frac{k}{2\pi}\right)^2 \int_{-\infty}^{\infty} W(\rho, \rho_d, z) \exp(-ik\theta \cdot \rho_d) d^2\rho_d, \quad (3)$$

where  $\rho' = (\rho'_1 + \rho'_2)/2$  and  $\rho'_d = (\rho'_1 - \rho'_2)$ ,  $\theta \equiv (\theta_x, \theta_y)$ , and  $k\theta_x$  and  $k\theta_y$  are the wave vector component along the  $x$  axis and  $y$  axis, respectively. On substituting from Eq. (2) together with Eq. (1) into Eq. (3), we obtain the WDF of PCDA beam at the source plane  $z=0$ , i.e.,

$$\begin{aligned} h(\rho', \theta, 0) &= \frac{k^2}{\pi} \sum_{t=1}^M \sum_{r=1}^M \sum_{s=1}^4 \frac{\alpha_t \alpha_r}{(-1)^{s+1} c_0} \\ &\quad \left[ 1 - 2\beta x' + \beta^2 x'^2 - \frac{\beta^2}{2c_0} - \frac{\beta^2}{c_0^2} (q_s x' + ik\theta_x)^2 \right] \\ &\quad \times \exp\left[ -p_s \rho'^2 + \frac{1}{c_0} (q_s \rho' + ik\theta)^2 \right], \end{aligned} \quad (4)$$

where  $p_1 = \frac{t}{w_0^2} + \frac{r}{w_0'^2}$ ,  $p_2 = \frac{t}{w_0^2} + \frac{r}{w_0'^2}$ ,  $p_3 = \frac{t}{w_0'^2} + \frac{r}{w_0^2}$ ,  $p_4 = \frac{t}{w_0'^2} + \frac{r}{w_0^2}$ ,  $q_1 = \frac{t}{w_0^2} - \frac{r}{w_0'^2}$ ,  $q_2 = \frac{t}{w_0^2} - \frac{r}{w_0'^2}$ ,  $q_3 = \frac{t}{w_0'^2} - \frac{r}{w_0^2}$ ,  $q_4 = \frac{t}{w_0'^2} - \frac{r}{w_0^2}$  and  $c_0 = p_s + \frac{2}{\sigma_0^2}$ .

The moment of the order  $n_1 + n_2 + m_1 + m_2$  of the WDF is given by [29]

$$\langle x^{n_1} y^{n_2} \theta_x^{m_1} \theta_y^{m_2} \rangle = \frac{1}{P} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^{n_1} y^{n_2} \theta_x^{m_1} \theta_y^{m_2} h(\rho, \theta, z) d^2\rho d^2\theta, \quad (5)$$

where  $P = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\rho, \theta, z) d^2\rho d^2\theta$  is the total power of the light beam.

On substituting from Eq. (4) into Eq. (5), and recalling the integral formula, i.e., [30]

$$\int_{-\infty}^{\infty} x^n \exp(-ax^2 + 2bx) dx = \frac{1}{2^{n-1} a} \sqrt{\frac{\pi}{a}} \frac{d^{n-1}}{db^{n-1}} \left[ \text{bexp}\left(\frac{b^2}{a}\right) \right] \quad (a > 0), \quad (6)$$

after tedious integral calculations, we can obtain the higher-order moments of PCDA beams along the  $x$  direction at the source plane  $z=0$ , which are given as follows.

### 2.1. First-order moments

$$\langle x_0 \rangle = -\frac{\pi\beta}{P} \sum_{t=1}^M \sum_{r=1}^M \sum_{s=1}^4 \frac{\alpha_t \alpha_r}{(-1)^{s+1} p_s^2}, \quad (7a)$$

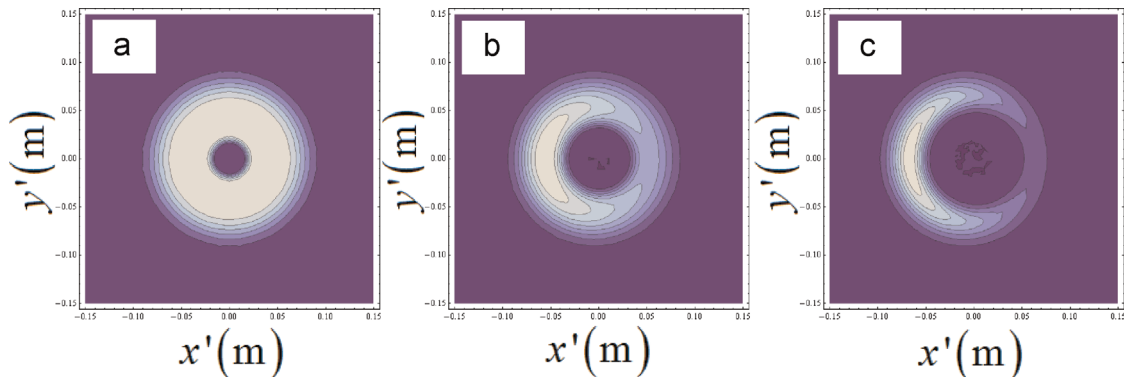


Fig. 1. Counter-lines of the intensity of PCDA beams at the source plane  $z=0$ . (a)  $\beta=0 \text{ m}^{-1}$ ,  $\varepsilon=0.2$ ; (b)  $\beta=3 \text{ m}^{-1}$ ,  $\varepsilon=0.4$ ; (c)  $\beta=6 \text{ m}^{-1}$ ,  $\varepsilon=0.6$ .

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