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Coherence of a near diffraction limited undulator synchrotron radiation source

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ABSTRACT

We present experimental measurements of the coherence of an undulator synchrotron radiation source near to the diffraction limit condition. These measurements have been done following two objectives. The first one is to verify a fundamental point of the theory of synchrotron radiation. To our knowledge, since it has been re-written by Geloni et al. [1], no experimental verification to validate the theory has been performed. Our measurement proves the theory to be valid for the case of a near diffraction limited undulator source. The other objective is to measure the coherence of the synchrotron radiation on the I05 beamline of Diamond, in front of the last focusing element of the beamline. This knowledge can be used to predict the focusing performance of the beamline by means of Fourier Optics.

1. Introduction

Third generation synchrotron light sources are now evolving towards storage rings with emittances ranging from $\epsilon = [10, 500]$ pm rad. For instance, emittance of the MAX IV synchrotron light source [2], soon to be commissioned, will be 240 pm rad. Similarly, a proposed project on the decommissioned PEP presents a 12 pm rad emittance [3]. Many other projects, planned or proposed, either using an existing facility or building a new one, are proposing new lattices based on multi-bend achromats with the purpose of having an ultra small emittance. The undulator Synchrotron Radiation (SR) will be at the diffraction limit for wavelengths satisfying $(\lambda/2\pi)$âª¢ϵ [4]. So the smaller the emittance, the smaller the undulator SR wavelength at which the diffraction limited is reached. In this paper, important results on the coherence properties of undulator SR are stated. To our knowledge, these properties of undulator SR for third generation light source operating near or at the diffraction limit have not been experimentally verified; in particular, the non-applicability of the van Cittert Zernike (VCZ) theorem [5] for the case of a source at the diffraction limit. We will verify experimentally that this is effectively the case: for a source at near diffraction limit, the VCZ is not applicable. In this paper, we present measurement of the transverse coherence of an undulator SR source at near the diffraction limit, by means of a Young interferometer [6,7]. The undulator is a quasi-harmonic 5 m long Helical undulator, designed for VUV to soft X-ray ARPES experiments on the I05 beamline of Diamond Light Source. Beamlines of this kind are used for photoemission spectroscopy experiments [8], where the coherence of the light is not exploited. In the near future, however, the beamline will be equipped with a Fresnel Zone Plate focusing optic whose properties can be modeled only with a good knowledge of the coherence. This paper is organized as follows. In Section 2, we will recall the theory with the same notation as used in [1]. In Section 4 we will present the experimental setup and condition as described with the theoretical notation, showing that the source is fully at the diffraction limit in the vertical plane, but only near the diffraction limit in the horizontal plane. In Section 5 we will show the experimental results, and compare them to both the theory of an undulator SR from statistical optics as described in [1] and prediction from the VCZ theorem. Finally, we will discuss the results before closing on concluding remarks.

2. SR coherence: theoretical background

As shown in [1], undulator SR, its cross spectral density[1] and associated spectral degree of coherence, in a general case, are

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[1] The cross spectral density is given by the cross correlation of the power spectrum of two given signals.

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described from statistical optics. Here we simply recall these expressions as given in [1] in their normalized units.

The cross spectral density, in normalized units is given by (Eq. 38 of [1])

$$\hat{G} = \left(\frac{2c^2\gamma}{K\omega e A_{jj}}\right)^2 G \tag{1}$$

with G being the cross spectral density, c the speed of light; $K = \bar{\lambda}_\omega e H_w / m_e c^2$ the undulator deflection parameter, with e being the electron charge and m_e the electron mass, H_w is the undulator magnetic field, and $\bar{\lambda}_\omega$ the undulator period; $\omega = 2\pi c/\lambda$ is the radial frequency of the SR field at which the cross spectral density is observed, and λ its associated wavelength; $A_{jj} = J_0(\xi) - J_1(\xi)$, with J_i being the modified Bessel function at order i and with the argument $\xi = K^2/(4 + 2K^2)$.

$\hat{G}$ is given by (Eq. 43 of [1] in which we have also introduced the integration over the energy distribution)

$$\begin{aligned}\hat{G}\left(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\hat{\theta}}_1, \vec{\hat{\theta}}_2\right) = e^{i(\hat{z}/2)\left(\vec{\hat{\theta}}_1^{\,2} - \vec{\hat{\theta}}_2^{\,2}\right)} &\int d\hat{\xi}_E \int d\vec{\hat{l}} \int d\vec{\hat{\eta}}\, f_E\left(\hat{\xi}_E\right) \\ &f_\perp\left(\vec{\hat{l}}, \vec{\hat{\eta}}\right) \\ &e^{i\left(\vec{\hat{\theta}}_1 - \vec{\hat{\theta}}_2\right)\cdot\vec{\hat{l}}}\, \Psi_C\left(\hat{z}, \hat{C} + \hat{\xi}_E, \left|\vec{\hat{\theta}}_1 - \frac{\vec{\hat{l}}}{\hat{z}} - \vec{\hat{\eta}}\right|\right) \\ &\Psi_C^*\left(\hat{z}, \hat{C} + \hat{\xi}_E, \left|\vec{\hat{\theta}}_2 - \frac{\vec{\hat{l}}}{\hat{z}} - \vec{\hat{\eta}}\right|\right)\end{aligned} \tag{2}$$

with

$$\hat{z} = \frac{z}{L_w} \tag{3a}$$

$$\hat{C} = L_w C = 2\pi N_w \frac{\omega - \omega_r}{\omega_r} \tag{3b}$$

$$L_w = N_w \lambda_w \tag{3c}$$

$$\vec{\hat{l}} = \vec{l}\sqrt{\frac{\omega}{L_w c}} \tag{3d}$$

$$\vec{\hat{\eta}} = \vec{\eta}\sqrt{\frac{\omega L_w}{c}} \tag{3e}$$

$$\vec{\hat{r}} = \vec{r}\sqrt{\frac{\omega}{L_w c}} \tag{3f}$$

$$\vec{\hat{\theta}} = \frac{\vec{\hat{r}}}{\hat{z}} \tag{3g}$$

$$\Delta\vec{\theta} = \vec{\hat{\theta}}_1 - \vec{\hat{\theta}}_2 \tag{3h}$$

$$\vec{\theta} = \frac{\vec{\hat{\theta}}_1 + \vec{\hat{\theta}}_2}{2}, \tag{3i}$$

which describes the normalized distance z to the undulator length, Eq. (3a); the normalized photon energy difference with respect to the undulator photon energy resonance, $(\omega - \omega_r)$, Eq. (3b); the undulator length L_w and number of undulator periods N_w; the particle transverse position with respect to the propagation axis z, Eq. (3d), and normalized angular momentum, Eq. (3e); the observation normalized position, Eq. (3f) and angle, Eq. (3g).

The last two equations introduce normalized variables which will be used later: the normalized difference and mean observation angles, Eqs. (3h) and (3i) respectively, between the two observation points at which $\hat{G}$ is evaluated.

So as described in [1], the normalized cross spectral density is evaluated by means of an integration over the bunch phase-space distribution in position, angle and energy given by the function $f_\perp$ (position and angle) and $f_{\hat{\xi}_E}$ (energy). Assuming phase-space distributions to be Gaussian and decoupled, which is a reasonable approximation for 3rd generation synchrotron light sources such as Diamond, they can be expressed in terms of dimensionless parameters as follows:

$$f_E\left(\hat{\xi}_E\right) = \frac{1}{\sqrt{2\pi}\,\sigma_E} e^{\hat{\xi}_E^2/2\hat{\sigma}_E^2} \tag{4}$$

with $\hat{\sigma}_E = 4\pi N_w \sigma_\epsilon$, and σ_ϵ the relative beam energy spread; and with the beam spatial and momentum distributions:

$$f_\perp\left(\vec{\hat{l}}, \vec{\hat{\eta}}\right) = \frac{1}{4\pi^2\sqrt{\hat{N}_x \hat{N}_y \hat{D}_x \hat{D}_y}} e^{-\left(\hat{l}_x^2/2\hat{N}_x\right) - \left(\hat{l}_y^2/2\hat{N}_y\right) - \left(\hat{\eta}_x^2/2\hat{D}_x\right) - \left(\hat{\eta}_y^2/2\hat{D}_y\right)} \tag{5}$$

with their r.m.s. values defined as

$$\hat{N}_{x,y} = \frac{2\pi\sigma_{x,y}^2}{\lambda L_w} \tag{6}$$

and

$$\hat{D}_{x,y} = \frac{2\pi\sigma_{x',y'}^2}{\lambda / L_w} \tag{7}$$

and $\sigma_{x,y}$ being the transverse beam size at the center of the undulator, and $\sigma_{x',y'}$ its divergence in the horizontal and vertical axis respectively.

The universal function Ψ_C is defined by Eq. 26 of [1]

$$\Psi_C(z, C, \alpha) \equiv \int_{-1/2}^{1/2} \frac{dz'}{z - z'} e^{i\left[Cz' + zz'\alpha^2/2(z - z')\right]} \tag{8}$$

For $\hat{z}$ â‪¢ 1 Eq. (8) turns to be a Sinus Cardinal[2] (Eq. 29 [1]).

The function $\hat{G}$ can be used to evaluate the spectral degree of coherence g as follows:

$$g\left(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\theta}, \Delta\vec{\theta}\right) = \frac{\hat{G}\left(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\theta}, \Delta\vec{\theta}\right)}{\hat{G}\left(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\theta} + \frac{\Delta\vec{\theta}}{2}, 0\right)^{1/2} \hat{G}\left(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\theta} - \frac{\Delta\vec{\theta}}{2}, 0\right)^{1/2}} \tag{9}$$

where $\hat{G}(\hat{\sigma}_E, \hat{z}, \hat{C}, \vec{\hat{\theta}}_i, 0)^{1/2}$ represents the normalized intensity[3]

[2] Sinus Cardinal is the function $\mathrm{sinc}(x) = \sin(x)/(x)$.

[3] The cross spectral density measured at two overlapping points is simply the Fourier transform of the auto correlation, in other words, the power spectrum or intensity at the observation point and at the observed frequency.