Optics Communications 359 (2016) 44–48

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Position-dependent oscillated decay of a two-level atom immersed in a two-dimensional photon fluid

Xiongfeng Zhang, Miao Yin*, Wenyao Liang

School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510640, People's Republic of China

A R T I C L E I N F O

Article history:
Received 13 June 2015
Received in revised form
18 September 2015
Accepted 19 September 2015

Keywords:
Spontaneous emission
Photon superfluid
Decay rate

A B S T R A C T

A Weisskopf–Wigner theory has been used to investigate the spontaneous emission of a two-level atom placed in a photon superfluid. It is found that the atom decays exponentially. However, the atomic decay rate changes periodically with the position of the atom and it is minimal when the atom is located at the wave nodes. The largest decay rate of the atom in photon superfluid has the same order of magnitude as it is in vacuum of free space. Moreover, the analytical result shows that the decay of an atom in photon superfluid, compared with that in planar cavity without photon superfluid, will be inhibited. The physical origin of atomic decay inhibition is also discussed.

1. Introduction

In recent years, much attention has been paid to study spontaneous emission [1–4], which arises from the interaction of an atom with the vacuum field modes. It has been long recognized that when an atom is placed in a new environment instead of the vacuum of free space, the spontaneous emission can be modified. There has been substantial research on spontaneous emission of atoms placed in different electromagnetic environments, such as photonic crystals, Kerr nonlinear blackbody, and microcavities [5–9]. Generally speaking, it has been shown that spontaneous emission can be suppressed or enhanced in a cavity [10–12].

An interesting environment called photon superfluid, a new state of light, has attracted growing attention of scientists. Photon superfluid is a manifestation of Bose–Einstein condensation and is observed firstly by Klaers et al. [13] by using a macroscopic optical cavity containing a dye solution. Thereafter, a large amount of papers concerning photon appears both theoretically [14–17] and experimentally [18,19]. In order to create a stable luminous fluid, it is crucial to give a finite effective mass to the photon. A simple strategy for this purpose involves a spatial confinement of the photons by metallic or dielectric planar mirrors [20]. A first elaboration of the concept of photon superfluid dates back to the work of Brambilla et al. [21] and Staliunas [22]. In such an environment almost all photons occupy the ground state. Compared with usual electromagnetic environment, photon in superfluid is not three but two dimensional. Therefore, it is interesting to study the spontaneous emission of atoms immersed in this new photonic state. In this paper, we aim to investigate spontaneous emission of a two-level atom placed in a photon superfluid. The atomic decay rate is obtained by using quantum field theory. It is found that an atom placed in different positions decays in different rates. Concretely, its decay rate is very small (almost tends to zero) when it is located at the wave nodes. Meanwhile, the order of magnitudes of the maximum of the depopulation is the same as the one in vacuum of free space. Moreover, the analytical result shows that the decay of an atom located at photon superfluid, compared with that at the planar cavity without superfluid, will be inhibited.

This paper is structured as follows. First, we establish a model for photon superfluid of two-dimensional photon gas. Subsequently, we use the proper vector potential between two metal mirrors to deduce the Hamiltonian of the atom-photon system. Then we investigate the time-evolution properties of the two-level atom. A summary is given at last.

2. Microcavity resonator and Hamiltonian of two-dimensional photon fluid

It is well known that in normal blackbody, the chemical potential of the photon system is vanishing. In other words, the number of photons will change with temperature. However, one of the preconditions of BEC is that the chemical potential should not be vanished. Therefore, for blackbody radiation, the photons will disappear in the cavity walls instead of occupying the ground state. In 1999, in order to solve these problems and get a photon superfluid, Raymond Y. Chiao et al. [23] theoretically established a model of photon superfluid by confining photons in two planar

* Corresponding author. Fax: +86 20 87113934.
E-mail address: scmyin@scut.edu.cn (M. Yin).

http://dx.doi.org/10.1016/j.optcom.2015.09.064

mirrors. The distance between mirrors is small enough to make the longitudinal modes of photons frozen out. Subsequently, the photons are effectively two-dimensional. In their theory, almost all of the photons condense at the ground state and the chemical potential does not vanish.

In this paper, following Raymond Y. Chiao et al. [23], we consider photons are confined in two planar mirrors. Because the frequency spacing between adjacent longitudinal modes is large enough, it is appropriate to assume that longitudinal mode of photons is frozen out and their transverse modes remain free, which makes the photon fluid effectively two-dimensional. At low temperature, photons condense into ground state. However, there is a little depletion due to the weak interaction between photons. Precisely speaking, there may produce photons with momentum $\boldsymbol{\kappa}$ and $-\boldsymbol{\kappa}$ respectively during two zero-momentum photon interaction.

Because the longitudinal mode of photons $k_z = \frac{n\pi}{L}$ is frozen, where n is a fixed integer and L is the distance between mirrors, it is reasonable to assume that $\sqrt{k_x^2 + k_y^2} \ll k_z$ (it implies that $\sqrt{p_x^2 + p_y^2} \ll p_z$) and rewrite the energy of photon to be

$$E(p) = c\,[p_x^2 + p_y^2 + p_z^2]^{1/2} \cong mc^2 + \frac{p_T^2}{2m}, \tag{1}$$

where $m = \frac{\hbar n\pi}{Lc}$ is the effective mass of two-dimensional photon, $\boldsymbol{p}_T = (p_x, p_y)$ is two-dimensional momentum and $p_T = \sqrt{p_x^2 + p_y^2}$. By redefining the zero of the energy, we could only consider the effective kinetic energy $\varepsilon(\boldsymbol{\kappa}) = \frac{(\hbar\kappa)^2}{2m}$, where $\boldsymbol{\kappa} = \frac{\boldsymbol{p}_T}{\hbar}$.

Following Raymond Y. Chiao's work [23] and reference [20], the Hamiltonian of the field can be expressed as

$$H_F = \sum_{\boldsymbol{\kappa},\lambda} \varepsilon(\boldsymbol{\kappa}) a_{\boldsymbol{\kappa}}^{\lambda\dagger} a_{\boldsymbol{\kappa}}^{\lambda} + \frac{1}{2} \sum_{\boldsymbol{\kappa}qs,\lambda1,\lambda2} V(\boldsymbol{\kappa}) a_{\boldsymbol{s}+\boldsymbol{\kappa}}^{\lambda1\dagger} a_{\boldsymbol{q}-\boldsymbol{k}}^{\lambda2\dagger} a_{\boldsymbol{s}}^{\lambda1} a_{\boldsymbol{q}}^{\lambda2} - \mu \sum_{\boldsymbol{\kappa},\lambda} a_{\boldsymbol{\kappa}}^{\lambda\dagger} a_{\boldsymbol{\kappa}}^{\lambda}, \tag{2}$$

where $a_{\boldsymbol{\kappa}}^{\lambda\dagger}$ and $a_{\boldsymbol{\kappa}}^{\lambda}$ are the creation and annihilation operators of photons with momentum $\hbar\boldsymbol{\kappa}$ and polarization λ ($\lambda = E, M$ represents the two types of polarization), respectively. And they satisfy the Bose commutation relations

$$[a_{\boldsymbol{\kappa}}^{\lambda1}, a_{\boldsymbol{q}}^{\lambda2\dagger}] = \delta_{\boldsymbol{\kappa},\boldsymbol{q}}\delta_{\lambda1,\lambda2},\ [a_{\boldsymbol{\kappa}}^{\lambda1}, a_{\boldsymbol{q}}^{\lambda2}] = [a_{\boldsymbol{\kappa}}^{\lambda1\dagger}, a_{\boldsymbol{q}}^{\lambda2\dagger}] = 0. \tag{3}$$

The first term of the Hamiltonian represents the energy of the free photon system and the second term represents the interactional energy $V(\boldsymbol{\kappa})$ between photons. Because the system is open and the photons will get lost, we should allow the number of photons fluctuates around a constant value. A standard way to describe this is to use Lagrange multiplier [15], specifically, the corresponding Hamiltonian is $-\mu \sum_{\boldsymbol{\kappa},\lambda} a_{\boldsymbol{\kappa}}^{\lambda\dagger} a_{\boldsymbol{\kappa}}^{\lambda}$, with μ being the chemical potential.

At a very low temperature, there exists a BEC and we denote the number of photons with zero momentum as N_0, here N_0 is big enough. Since the interaction is weak, we shall suppose that N_0 will remain the same number. Actually, in the experiment, N_0 depends on the intensity of the incident laser beam. Now let us consider the case that zero momentum operators $a_0^\dagger$ and a_0 act on the ground state $|\psi_0(N_0)\rangle$:

$$a_0|\psi_0(N_0)\rangle = \sqrt{N_0}\,|\psi_0(N_0 - 1)\rangle$$
$$a_0^\dagger|\psi_0(N_0)\rangle = \sqrt{N_0 + 1}\,|\psi_0(N_0 + 1)\rangle \cong \sqrt{N_0}\,|\psi_0(N_0 + 1)\rangle. \tag{4}$$

Here we use the fact that $N_0 \gg 1$. The above result implies that when acting on the zero momentum state, both $a_0^\dagger$ and a_0 could be regarded as a c-number $\sqrt{N_0}$. Therefore, the total Hamiltonian can be rewritten as

$$H_F \approx \varepsilon'(0) + \sum_{\boldsymbol{\kappa},\lambda1,\lambda2 \neq 0} \varepsilon'(\boldsymbol{\kappa}) a_{\boldsymbol{\kappa}}^{\lambda1\dagger} a_{-\boldsymbol{\kappa}}^{\lambda2} + \sum_{\boldsymbol{\kappa}\neq 0,\lambda1,\lambda2} N_0 V(\boldsymbol{\kappa}) (a_{\boldsymbol{\kappa}}^{\lambda1\dagger} a_{-\boldsymbol{\kappa}}^{\lambda2\dagger} + a_{\boldsymbol{\kappa}}^{\lambda1} a_{-\boldsymbol{\kappa}}^{\lambda2}), \tag{5}$$

where $\varepsilon'(0) = N_0\varepsilon(0) + \frac{1}{2}V_0 N_0^2$ and $\varepsilon'(\boldsymbol{\kappa}) = \varepsilon(\boldsymbol{\kappa}) + N_0 V(\boldsymbol{\kappa})$. Here the interaction term between two non-zero momentum photons has been abandoned. And we obtain

$$\mu = \frac{\partial\langle\psi_0|H|\psi_0\rangle}{\partial N} \cong \frac{\partial \frac{1}{2}N^2 V(0)}{\partial N} = NV(0) \approx N_0 V(0). \tag{6}$$

In order to obtain the quadratic-form Hamiltonian, following Bogoliubov [24,25], we make a canonical transformation

$$b_{\boldsymbol{\kappa}}^{\lambda} = u_{\kappa} a_{\boldsymbol{\kappa}}^{\lambda} + v_{\kappa} a_{-\boldsymbol{\kappa}}^{\lambda\dagger}$$
$$b_{\boldsymbol{\kappa}}^{\lambda\dagger} = u_{\kappa} a_{\boldsymbol{\kappa}}^{\lambda\dagger} + v_{\kappa} a_{-\boldsymbol{\kappa}}^{\lambda}, \tag{7}$$

where $b_{\boldsymbol{\kappa}}^{\lambda}$ and $b_{\boldsymbol{\kappa}}^{\lambda\dagger}$ are respectively the annihilation and creation operators of the quasi-particles. Furthermore, u_κ and v_κ obey the relation $u_\kappa^2 - v_\kappa^2 = 1$, which implies the Bose commutation relations of $b_{\boldsymbol{\kappa}}^{\lambda}$ and $b_{\boldsymbol{\kappa}}^{\lambda\dagger}$,

$$[b_{\boldsymbol{\kappa}}^{\lambda1}, b_{\boldsymbol{q}}^{\lambda2\dagger}] = \delta_{\boldsymbol{\kappa},\boldsymbol{q}}\delta_{\lambda1,\lambda2} \quad \text{and} \quad [b_{\boldsymbol{\kappa}}^{\lambda1}, b_{\boldsymbol{q}}^{\lambda2}] = [b_{\boldsymbol{\kappa}}^{\lambda1\dagger}, b_{\boldsymbol{q}}^{\lambda2\dagger}] = 0. \tag{8}$$

Specifically, the diagonal form Hamiltonian of Eq. (5) can be written as

$$H_F = \sum_{\boldsymbol{\kappa},\lambda} \hbar\tilde{\omega}_{\boldsymbol{\kappa}} \left(b_{\boldsymbol{\kappa}}^{\lambda\dagger} b_{\boldsymbol{\kappa}}^{\lambda} + \frac{1}{2} \right) + \text{const}, \tag{9}$$

here $\hbar\tilde{\omega}_{\boldsymbol{\kappa}}$ represents the energy of a quasi-particle with momentum $\hbar\boldsymbol{\kappa}$. After substituting Eq. (7) into Eq. (5) and comparing the result with the diagonal form Eq. (9), we get the condition for diagonalization

$$\begin{cases} \hbar\omega_{\boldsymbol{\kappa}} u_\kappa v_\kappa = \frac{1}{2} N_0 V(\boldsymbol{\kappa}), \\ u_\kappa^2 = \frac{1}{2}\left[1 + \frac{\varepsilon'(\boldsymbol{\kappa})}{\hbar\omega_{\boldsymbol{\kappa}}}\right], \\ v_\kappa^2 = \frac{1}{2}\left[-1 + \frac{\varepsilon'(\boldsymbol{\kappa})}{\hbar\omega_{\boldsymbol{\kappa}}}\right]. \end{cases} \tag{10}$$

By solving Eq. (10), it is easy to obtain Bogoliubov dispersion relation

$$\hbar\tilde{\omega}_{\boldsymbol{\kappa}} = \sqrt{\frac{(\hbar\kappa)^2 N_0 V(\boldsymbol{\kappa})}{m} + \frac{(\hbar\kappa)^4}{4m^2}}. \tag{11}$$

3. Interaction between the field and two-level atom

Here, we consider a two-level atom immersed in the photon fluid, as seen in Fig. 1. Let $|a\rangle$ being its high-energy eigenstate with

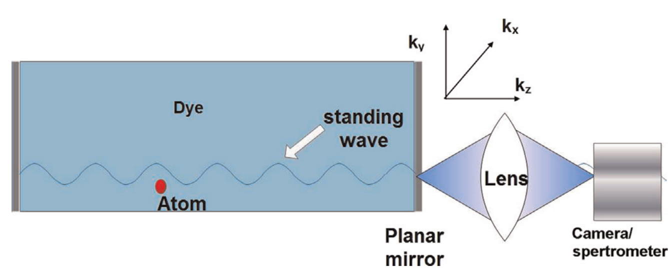

Fig. 1. Schematic of the optical microcavity. The cavity consists of two planar mirrors which is fulfilled with dye solution. An atom is immersed inside the microcavity.