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Nonclassical properties of coherent light in a pair of coupled anharmonic oscillators

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ABSTRACT

The Hamiltonian and hence the equations of motion involving the field operators of two anharmonic oscillators coupled through a linear one is framed. It is found that these equations of motion involving the non-commuting field operators are nonlinear and are coupled to each other and hence pose a great problem for getting the solutions. In order to investigate the dynamics and hence the nonclassical properties of the radiation fields, we obtain approximate analytical solutions of these coupled nonlinear differential equations involving the non-commuting field operators up to the second orders in anharmonic and coupling constants. These solutions are found useful for investigating the squeezing of pure and mixed modes, amplitude squared squeezing, principal squeezing, and the photon antibunching of the input coherent radiation field. With the suitable choice of the parameters (photon number in various field modes, anharmonic, and coupling constants, etc.), we calculate the second order variances of field quadratures of various modes and hence the squeezing, amplitude squared, and mixed mode squeezing of the input coherent light. In the absence of anharmonicities, it is found that these nonlinear nonclassical phenomena (squeezing of pure and mixed modes, amplitude squared squeezing and photon antibunching) are completely absent. The percentage of squeezing, mixed mode squeezing, amplitude squared squeezing increase with the increase of photon number and the dimensionless interaction time. The collapse and revival phenomena in squeezing, mixed mode squeezing and amplitude squared squeezing are exhibited. With the increase of the interaction time, the monotonic increasing nature of the squeezing effects reveal the presence of unwanted secular terms. It is established that the mere coupling of two oscillators through a third one does not produces the squeezing effects of input coherent light. However, the pure nonclassical phenomena of antibunching of photons in vacuum field modes are obtained through the mere coupling and hence the transfers of photons from the remaining coupled mode.

1. Introduction

The availability of high power laser sources is found extremely useful in the development of nonlinear optics, laser spectroscopy and quantum optics. In particular, during the last three decades, we find tremendous progresses in the field of quantum optics. These include the generation of squeezed states [1,2] and photon antibunching of the radiation field [3], etc. According to Heisenberg, there are always some fluctuations involving the measurements of canonically conjugate variables and their product obeys the famous uncertainty relation named after him. Barring the ground state of the harmonic oscillator, the uncertainty product of canonically conjugate variables attains the minimum value for coherent state only. It is established that the fluctuation of the dimensionless quadrature components involving the coherent state is equal and is termed as zero point fluctuations [4]. Before the discovery of squeezed states, the ZPF was regarded as the standard quantum limit (SQL). The squeezed state is a quantum state of the radiation field in which one of the quadrature fluctuations goes below the ZPF and hence the SQL [5]. Of course, the uncertainty relation due to Heisenberg is to be respected for the squeezed state as well. The abstract idea of squeezed state came into reality through the remarkable experiment in the degenerate four wave mixing of sodium atom [6]. Till now, we find lot of experimental and theoretical developments involving the researches in squeezed state [1,2,7–11]. It is because the squeezed state could be used in the detection of gravitational wave and noise free communication [1,2]. It is claimed that the sensitivity of the LIGO (Laser Interferometer Gravitational wave Observatory) detector is increased in an unprecedented way by using the

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squeezed state. In addition to the applications of squeezed states in gravitational wave detection [9], it is further used in the demonstration of entangled state and hence the quantum teleportation [12] and in the high precision measurements and meteorology [13,14]. The essence of these researches is based on the nonlinear matter-field interactions. In the theoretical investigation of the squeezing effects of the radiation field, we calculate the second order variances involving the field (quadrature) operators. The non-vanishing second order variances (ZPF in terms of the initial coherent state) follow from the non-commutative nature of the quadrature operators. Therefore, it appears that the nonlinear interaction and hence the additional parameters are essential to manipulate the second order variances and hence the squeezed state. Keeping in mind of second order variances and hence the squeezed states, people have generalized for higher order variances and hence the higher ordered squeezed state. Higher order squeezing in terms of the field amplitude has also been developed [15]. A close look to these investigation of squeezing, higher order squeezing and in the higher ordered squeezing involving the amplitudes reveals that the interaction basically nonlinear in nature. As a matter of fact, it is normally believed that the nonlinear interaction involving field operators is necessary to invoke the squeezing effects. It may be accounted because the linear interaction does not provide any significant contribution to the second and higher order variances involving the quadrature operators.

Of late, we observe the huge development in the investigation of the quantum properties of micro- or nano-mechanical systems [16–25]. It is because of the availability of ultra-cooled mechanical system which were otherwise impossible before the advent of laser cooling. Depending upon the various situations and purposes, these nanomechanical systems are fabricated. Because of the complex nature, it is not always possible to provide theoretical analysis of all these systems. We, therefore, seek some relevant simple nanomechanical systems which are very close to the physical situations. For example, the model of two anharmonic oscillators coupled through a third oscillator is found useful in the context of quantum entanglement between various modes and also from the quantum optical applications. The model is potentially useful for further investigation on squeezing, higher-ordered squeezing, mixed mode squeezing and the antibunching of various field modes of the radiation fields. The present paper is thus aimed for a complete presentation of the quantum optical properties of two anharmonic oscillators coupled through a third oscillator.

2. The model Hamiltonian

We consider two two-photon anharmonic oscillators of field frequency ω_0. These oscillators are coupled to a linear oscillator of field frequency ω. In order to simplify the situation, we consider that the anharmonic constant β of the two nano-anharmonic oscillators are same. Nevertheless, the anharmonic terms are chosen so as to remove the nonconserving energy terms. In other words, the anharmonic terms are diagonalizable in the Fock state basis. The coupling parameter k is of comparable magnitude with those of β. Therefore, the present investigation takes care the situation of a weak coupling condition. The field operators $a_1(a_1^\dagger)$, $a_2(a_2^\dagger)$ and $a_3(a_3^\dagger)$ correspond to the annihilation (creation) operators involving the first anharmonic oscillator, second anharmonic oscillator and the linear oscillator respectively. Therefore, the Hamiltonian of the system follows as [22,23]

$$H = \hbar\omega_0\left(a_1^\dagger a_1 + a_2^\dagger a_2\right) + \hbar\omega a_3^\dagger a_3 + \hbar\beta\left(a_1^{\dagger 2} a_1^2 + a_2^{\dagger 2} a_2^2\right) + \hbar k\left(a_1^\dagger + a_2^\dagger\right)a_3 + \hbar k^*\left(a_1 + a_2\right)a_3^\dagger \quad (1)$$

It is clear that the Hamiltonian (1) is quite useful from the view point of nanomechanical coupled anharmonic oscillators one and hence it attracts potential applications in quantum information theory. In fact, the present model can be thought of as a cavity with two modes which is a special case of a cavity of several modes [26,27]. The Hamiltonian (1) in its present form or in a slightly different form [28] might be useful in chemistry since it is established that the two C–H bonds of dihalomethane lead to the model of two quartic anharmonic oscillators coupled through the Jaynes–Cummings [29] type interaction. Of late, we find that the chain of coupled oscillator could be of use in utilizing the quantum state transfer [30–32]. Hence, it will be an interesting problem to investigate the effects of the coupling term and hence the quantum state transfer on the squeezing effects of the input coherent light involving three coupled oscillator (i.e. $\beta=0$ in (1)). In the absence of coupling (i.e. $k=0$), the Hamiltonian (1) corresponds three decoupled oscillators and hence an exactly solvable model.

The presence of nonlinear terms involving β and the coupling constant k in the model Hamiltonian (1) makes the model more general and a realistic one. In particular, the most of the problems of coupled oscillators are basically a special case of the present one under consideration. Of course, the presence of nonlinearity and the coupling terms poses a serious problems for analytical investigation. As a matter of fact, the presence of anharmonic terms are on the way of getting closed form analytical solutions of the problem. Nevertheless, we believe that the presence of anharmonic term might be useful for investigating the squeezed states, amplitude-squared squeezed states, photon antibunching and nonclassical photon statistics of the input radiation field. Therefore, in the present investigation, we would like to address few of the nonclassical properties of the input coherent light. Of course, we will provide approximate analytical solution to the model Hamiltonian (1).

2.1. The solution of field operators

The equations of motion involving the field operators corresponding to the Hamiltonian (1) are given by [23]

$$\begin{aligned} \dot{a}_1 &= -i\omega_0 a_1 - 2i\beta a_1^\dagger a_1^2 - ika_3 \\ \dot{a}_2 &= -i\omega_0 a_2 - 2i\beta a_2^\dagger a_2^2 - ika_3 \\ \dot{a}_3 &= -i\omega a_3 - ik(a_1 + a_2) \end{aligned} \quad (2)$$

where the coupling constant k is real. The solution of the above equations (2) is used in our recent article [23]. However, in the present communication, the detailed solutions are given. It will make the presentation self-consistent. Now, we have two special situations where these differential equations (2) and hence the model Hamiltonian (1) offers exact analytical solutions. First one, for $k=0$ (i.e. there is no coupling), three differential equations in (2) are completely decoupled. The corresponding solutions for the field operators are $a_1(t) = e^{-i(\omega_0+2\beta a_1^\dagger a_1)t}a_1(0)$, $a_2(t) = e^{-i(\omega_0+2\beta a_2^\dagger a_2)t}a_2(0)$, and $a_3(t) = e^{-i\omega t}a_3(0)$. Therefore, in case of mode a_1 and a_2, the field frequency ω_0 gets modified by the presence of Kerr-type nonlinearities involving the nonlinear constant β. The second situation for which the set of differential equations are exactly solvable is for $\beta=0$. The corresponding solutions may be obtained from our general solutions (3) by putting $\beta=0$. Therefore, all the three field modes a_1, a_2, and a_3 are decoupled and are hardly useful for the investigation of entanglement and other nonclassical properties of the radiation fields. On the other hand, the presence of both the nonlinear constant β and the coupling constant k make the equations (2) unsolvable in closed analytical forms. Therefore, one has to explore either numerical solutions or an approximate analytical solution for further investigations. It is true that the numerical