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Characterizing the optical chaos in a special type of small networks of semiconductor lasers using permutation entropy

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ABSTRACT

A critical issue in the usage of permutation entropy (PE) for the complexity measure is the selection of the embedding time delay. It is known that PE evaluated with multiple delays can help us gain additional insight into the unpredictability degree at different timescales. In this paper, we investigate the unpredictability degree of the optical chaos in a special type of small networks of semiconductor lasers (SLs) using permutation entropy. These laser systems, such as SLs subject to delayed self-feedback, mutually delay-coupled SLs, and a ring configuration consisting of three unidirectionally delay-coupled SLs, exhibit similar correlation properties characterized by autocorrelation function. The results show that, for a given embedding time delay, the curves for the PE follow the same trends upon monotonically increasing the coupling rate, regardless of how complex the system structure actually is. Furthermore, the results corroborate the effectiveness of PE as an unpredictability measure for small networks consisting of SLs with a single coupling delay, which can be extracted from the intensity autocorrelation function. Finally, we show an example for generating unpredictability-enhanced optical chaos based on small networks of SLs.

1. Introduction

Since the seminal work of Bandt and Pompe in 2002, permutation entropy (PE) [1] has been widely used to quantify the unpredictability of chaotic time series in the literature [2–4]. This unpredictability measure for time series relies on comparison of neighboring values and can be directly applied to any nonlinear dynamical systems. Interestingly, PE has been successfully applied to optoelectronic systems, e.g., chaotic semiconductor lasers (SLs). Rosso et al. have demonstrated that it is possible to detect the presence of small-amplitude message embedded in chaotic carriers via PE [5]. For SLs with delayed self-feedback [Fig. 1(a)], Zunino et al. have found that PE evaluated at specific time scales, feedback time delay, gives valuable information about the degree of unpredictability of the chaotic laser dynamics [6], in good agreement with the Kolmogorov–Sinai entropy; Soriano et al. have demonstrated that the estimation of PE is able to identify characteristic time scales present in the chaotic dynamics [7]; Toomey and Kane have obtained the PE maps of the dynamical unpredictability at different time scales [8]. Besides, Xiang et al. have reported the chaotic unpredictability of vertical-cavity surface-emitting lasers with polarized optical feedback based on the estimation of PE [9,10]. On the other hand, PE has been proposed to analyze time-delay signatures in the optical chaos [11,12].

Despite many papers in the literature deal with the validity of PE for quantifying the unpredictability degree of nonlinear time series, a comparison of a special type of small networks of SLs of Fig. 1 in terms of the estimation of PE has not yet been presented. These three configurations are of great interest, since the configuration in Fig. 1(a) is the simplest chaos generator, i.e., SLs with delayed self-feedback (a coupling time delay τ), and has been most widely investigated in the literature [6–8,13], while those in Figs. 1 (b) and (c) represent its variations, with the purpose of improving the quality of chaotic sources [14–17], where Fig. 1(b) corresponds to mutually delay-coupled SLs with a coupling time delay $\tau/2$ and (c) represents a ring configuration consisting of three unidirectionally delay-coupled SLs with a coupling time delay $\tau/3$. In fact, the dynamics and correlation properties of these three delay-coupled oscillators have been already studied (called small networks in this paper) [18,19], however, a systematical comparison of the unpredictability variations based on PE in this special type of small networks may be valuable for the potential applications of optical chaos, such as chaos-based communications and random bit generators [20–25].

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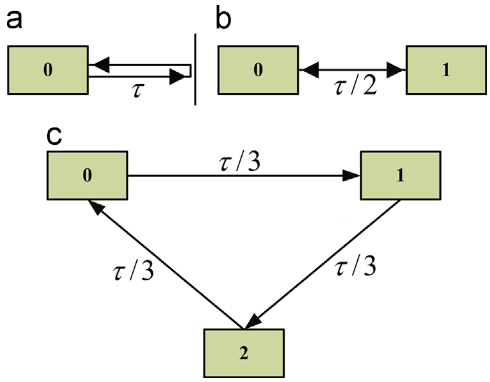

Fig. 1. (a) SL with delayed self-feedback, (b) two bidirectionally delay-coupled SLs, and (c) a ring configuration of three unidirectionally delay-coupled SLs. τ stands for feedback (coupling) time delay.

In the following, this point is addressed in detail. In Section 2, the Lang–Kobayashi (L–K) equations of the configurations shown in Fig. 1 are presented [26]. And PE is briefly introduced to quantitatively evaluate the unpredictability degree of chaotic signals. Section 3 is devoted to the numerical results of the systematical comparison. Finally, conclusions are given in Section 4.

2. Theory

2.1. Rate equations

For configuration in Fig. 1(a), the L–K model can be used, while for describing those in Figs. 1(b) and (c), the modified L–K model should be used, where the self-feedback term is replaced with external optical injection term.

The L–K rate equations for the complex slowly varying amplitude of the electric field $E(t)$ and the carrier number inside the cavity $N(t)$ for the laser can be given by [18,19]

$$\dot{E}_j(t) = \frac{(1+i\alpha)}{2}\left[G_j(t) - \frac{1}{\tau_p}\right]E_j(t) + \gamma E_k\left(t - \tau/M\right)e^{-i\Omega(\tau/M)}, \tag{1}$$

$$\dot{N}_j(t) = \frac{I}{e} - \frac{N_j(t)}{\tau_N} - G_j(t)\left|E_j(t)\right|^2, \tag{2}$$

where the index $j = 0, 1, 2$, $G_j(t) = g(N_j(t) - N_0)/(1 + s\left|E_j(t)\right|^2)$ is the optical gain of laser j, and Ω is the frequency of the free-running laser. For a single laser with self-feedback $(M = 1)$, $j = k = 0$; for mutually delay-coupled SLs $(M = 2)$, $(j, k) = (0, 1)$ and $(j, k) = (1, 0)$, respectively; for a ring configuration consisting of three unidirectionally delay-coupled SLs $(M = 3)$, $(j, k) = (0, 2)$, $(j, k) = (1, 0)$, and $(j, k) = (2, 1)$, respectively. The parameter $g = 1.5 \times 10^{-8}\ \text{ps}^{-1}$ is the differential gain coefficient and $s = 5 \times 10^{-7}$ is the saturation coefficient, $N_0 = 1.5 \times 10^8$ is the carrier density at transparency, $\alpha = 5$ is the linewidth-enhancement factor, $\tau_p = 2$ ps is the photon lifetime, $\tau_N = 2$ ns is the carrier lifetime, γ is the coupling rate and will be specified later, $\tau = 1$ ns is the feedback (coupling) time delay, and $J = 1.5J_{th}$ is the pump current (with J_{th} being the threshold current of the solitary laser). We assumed that the lasers involved have the identical parameters (e.g, the frequency detuning was not considered) and neglected the influence of noise.

The intensity time series of the laser was obtained by integrating Eqs. (1) and (2) using a fourth-order Runge–Kutta method with a time step of $\Delta t = 0.1$ ps. In our simulations, we followed the method introduced by Zunino [6], i.e., ten realizations were analyzed by taking into account the influence of statistical properties of the chaotic time series. Each one contains $N = 10^5$ data points with a sampling period of $\Omega_s = 1$ ps.

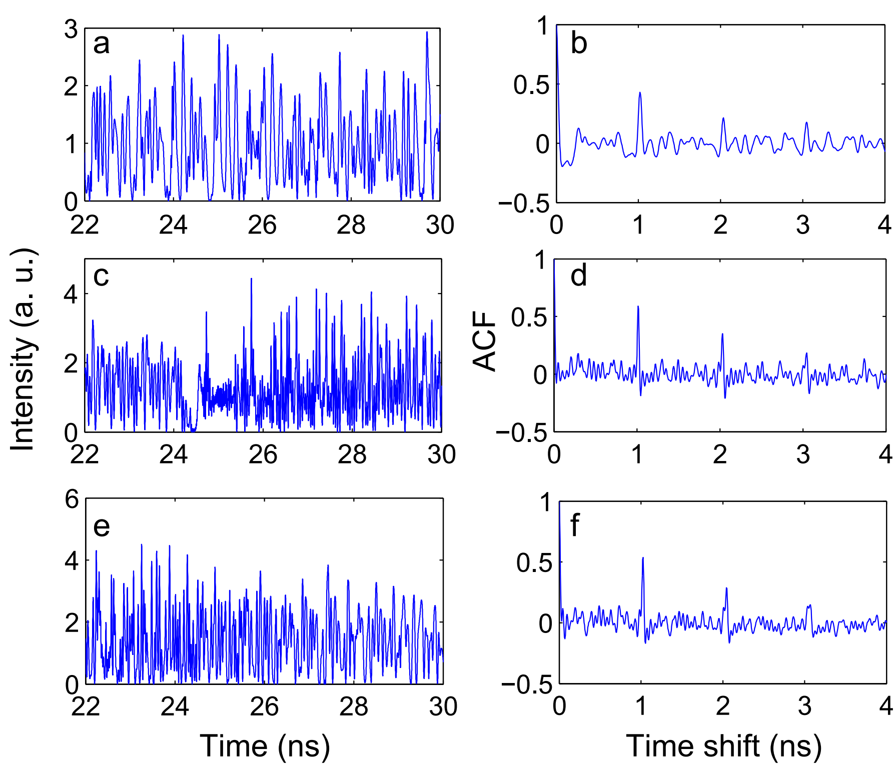

Fig. 2. Intensity time series (left column) and ACF plots (right column) for the SL with delayed self-feedback (first row), two bidirectionally delay-coupled SLs (second row), and a ring configuration of three unidirectionally delay-coupled SLs (third row). (a) and (b) $\gamma = 30\ \text{ns}^{-1}$; (c)–(f) $\gamma = 90\ \text{ns}^{-1}$.