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Nonparaxial propagation of a rectangular multi-Gaussian Schell-model beam

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ABSTRACT

According to the vectorial Rayleigh–Sommerfeld diffraction integral formulas, analytical expressions for the elements of 3×3 cross-spectral density matrix of polarized Rectangular Multi-Gaussian Schell-Model (RMGSM) beam's nonparaxial propagation in free space are derived, and the paraxial analytical expression has also been presented as a special case of nonparaxial propagation. Its statistical properties, including the distributions of spectral density, spectral degree of coherence, and spectral degree of polarization of nonparaxial RMGSM beam are numerically demonstrated. Results reveal that the statistical properties of RMGSM beam's nonparaxial propagation in free space are closely related to the initial beam's transverse width σ and correlation widths δ_x and δ_y. These characteristics of RMGSM beam might be useful in optical material surface processing, imaging, and free space optical communication.

1. Introduction

Since Gaussian Schell-Model (GSM) beam was firstly introduced by Wolf and Collett [1], researches about partially coherent beams have been intensively examined [2–9]. Recently, a novel named Rectangular Multi-Gaussian Schell-Model (RMGSM) was introduced by Korotkova [10], which is different from other Schell-Model class of partially coherent beams by virtue of its flat rectangular intensity distribution in the far field [11–14]. The unique feature of RMGSM beam has attracted a lot of attention, and researches about RMGSM beam were reported successively, including its propagation in atmospheric turbulence, scattering on a deterministic medium and fractional Fourier transforms, etc. [15–17].

As we know, for an electromagnetic beam propagation in free space, its electric vector E can be obtained by simplified calculation within the framework of classical electromagnetic theory. The premise of such simplification is paraxial approximation [18], namely, the longitudinal component E_z of electric vector is awfully small and could be ignored. However, the paraxial approximation is no longer valid for a beam with large divergence angle or small spot size that is of the order of light wavelength. In this case the electric vector E should be determined by strictly solving wave equations [18,19]. In this work, we use the vectorial Rayleigh–Sommerfeld diffraction integral formulas to solve the nonparaxial propagation of RMGSM beam and have derived the analytical expressions for the elements of 3×3 cross-spectral density matrix of RMGSM beam, and then its nonparaxial propagation properties would be investigated by numerical simulations. Results show that the distributions of spectral density, spectral degree of coherence and spectral degree of polarization of RMGSM beam's nonparaxial propagation in free space would be influenced by the initial beam's transverse width σ and correlation widths δ_x and δ_y along x and y directions. These characteristics of RMGSM beam might be useful in optical material surface processing, imaging, and free space optical communication.

2. Nonparaxial propagation of a RMGSM beam in free space

According to the vector Rayleigh–Sommerfeld diffraction integral formulas, nonparaxial propagation of a light beam in the half space $z > 0$ can be obtained, and each component of the vector beam in an arbitrary plane z can be expressed as [20]

$$E_x(x, y, z) = -\frac{1}{2\pi}\iint_{-\infty}^{\infty} E_x(x_0, y_0, 0)\frac{\partial \boldsymbol{R}(\boldsymbol{r}, \rho_0)}{\partial z}dx_0dy_0, \tag{1a}$$

$$E_y(x, y, z) = -\frac{1}{2\pi}\iint_{-\infty}^{\infty} E_y(x_0, y_0, 0)\frac{\partial \boldsymbol{R}(\boldsymbol{r}, \rho_0)}{\partial z}dx_0dy_0, \tag{1b}$$

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$$E_z(x, y, z) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \left(E_x(x_0, y_0, 0) \frac{\partial \boldsymbol{R}(\boldsymbol{r}, \rho_0)}{\partial x} + E_y(x_0, y_0, 0) \frac{\partial \boldsymbol{R}(\boldsymbol{r}, \boldsymbol{\rho_0})}{\partial y} \right) dx_0 dy_0, \tag{1c}$$

where $E_{x,y}(x_0, y_0, 0)$ are transverse components of the E vector in the input plane $z = 0$, and $E_{x,y,z}(x, y, z)$ are components of the E vector along x, y, and z directions in an arbitrary plane z, respectively. $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, $\rho_0 = x_0\boldsymbol{i} + y_0\boldsymbol{j}$, here $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors in x, y and z directions, respectively.

$$\boldsymbol{R}(\boldsymbol{r}, \rho_0) = \frac{\exp(ik|\boldsymbol{r} - \rho_0|)}{|\boldsymbol{r} - \rho_0|}, \tag{2}$$

here $k = 2\pi/\lambda$ is the wave number, and λ is the incident wavelength. When $|\boldsymbol{r} - \rho_0| >> \lambda$, $|\boldsymbol{r} - \rho_0|$ can be approximately expanded into [21]

$$|\boldsymbol{r} - \rho_0| \approx r + \frac{\rho_0^2 - 2xx_0 - 2yy_0}{2r}, \tag{3}$$

so Eq. (2) can be written as

$$\boldsymbol{R}(\boldsymbol{r}, \rho_0) = \frac{1}{r} \exp[ik(r + \frac{\rho_0^2 - 2xx_0 - 2yy_0}{2r})], \tag{4}$$

where $r = (x^2 + y^2 + z^2)^{1/2}$ and $\rho_0^2 = x_0^2 + y_0^2$.

On substituting from Eq. (4) into Eqs. (1a)–(1c), one obtains

$$E_x(x, y, z) = -\frac{iz}{\lambda r^2} \iint_{-\infty}^{\infty} E_x(x_0, y_0, 0) \times \exp[ik(r + \frac{\rho_0^2 - 2xx_0 - 2y_0 y}{2r})] dx_0 dy_0, \tag{5a}$$

$$E_y(x, y, z) = -\frac{iz}{\lambda r^2} \iint_{-\infty}^{\infty} E_y(x_0, y_0, 0) \times \exp[ik(r + \frac{\rho_0^2 - 2xx_0 - 2y_0 y}{2r})] dx_0 dy_0, \tag{5b}$$

$$E_z(x, y, z) = \frac{i}{\lambda r^2} \iint_{-\infty}^{\infty} [(x - x_0) E_x(x_0, y_0, 0) + (y - y_0) E_y(x_0, y_0, 0)] \times \exp[ik(r + \frac{\rho_0^2 - 2xx_0 - 2y_0 y}{2r})] dx_0 dy_0. \tag{5c}$$

Assuming that an incident field of RMGSM beam is polarized in the x direction, its cross-spectral density function in the source plane can be defined by [10]

$$W_{xx}^0(\rho_{10}, \rho_{20}, 0) = \exp(-\frac{\rho_{10}^2 + \rho_{20}^2}{4\sigma^2}) \cdot \frac{1}{C^2} \sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m-1}}{\sqrt{m}} \times \exp[-\frac{|x_{20} - x_{10}|^2}{2m\delta_x^2}] \times \sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m-1}}{\sqrt{m}} \exp[-\frac{|y_{20} - y_{10}|^2}{2m\delta_y^2}], \tag{6}$$

where $\rho_{10} = x_{10}\boldsymbol{i} + y_{10}\boldsymbol{j}$ and $\rho_{20} = x_{20}\boldsymbol{i} + y_{20}\boldsymbol{j}$ are two arbitrary transverse position vectors in the source plane, $C = \sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m-1}}{m}$ is the normalization factor with $\binom{M}{m}$ being binomial coefficients, M is the index of RMGSM beam, σ is the transverse beam width of the source, and δ_x and δ_y are the correlation widths of the source along x and y directions, respectively.

Let us consider a partially coherent beam of nonparaxial propagation whose second-order statistics properties may be characterized by the 3×3 cross-spectral density (CSD) matrix $\overleftrightarrow{W}(\rho_1, \rho_2, z)$ expressed as [22]

$$\overleftrightarrow{W}(\rho_1, \rho_2, z) = \begin{pmatrix} W_{xx}(\rho_1, \rho_2, z) & W_{xy}(\rho_1, \rho_2, z) & W_{xz}(\rho_1, \rho_2, z) \\ W_{xy}^*(\rho_1, \rho_2, z) & W_{yy}(\rho_1, \rho_2, z) & W_{yz}(\rho_1, \rho_2, z) \\ W_{xz}^*(\rho_1, \rho_2, z) & W_{yz}^*(\rho_1, \rho_2, z) & W_{zz}(\rho_1, \rho_2, z) \end{pmatrix}, \tag{7}$$

where, the element of the matrix is given by

$$W_{\alpha\beta}(\rho_1, \rho_2, z) = \left(E_\alpha^*(\rho_1, z) \cdot E_\beta(\rho_2, z) \right), (\alpha, \beta = x, y, z), \tag{8}$$

here $E_x(\rho, z)$, $E_y(\rho, z)$ and $E_z(\rho, z)$ represent the components of the random electric vector along x, y and z directions, respectively. The asterisk denotes the complex conjugate and the angular brackets stand for ensemble average.

Substituting Eqs. (5a)–(5c) into Eq. (8), one obtains the elements of 3×3 CSD matrix of the vector field in output plane:

$$W_{xx}(\rho_1, \rho_2, z) = \left(\frac{z}{\lambda}\right)^2 \cdot \frac{\exp[ik(r_2 - r_1)]}{r_1^2 r_2^2} \iint\iint_{-\infty}^{\infty} W_{xx}^0(\rho_{10}, \rho_{20}, 0) \times \exp\left(ik\frac{\rho_{20}^2 - 2x_2x_{20} - 2y_2y_{20}}{2r_2}\right) \times \exp\left(-ik\frac{\rho_{10}^2 - 2x_1x_{10} - 2y_1y_{10}}{2r_1}\right) dx_{10}dx_{20}dy_{10}dy_{20}, \tag{9}$$

$$W_{xz}(\rho_1, \rho_2, z) = -\frac{z}{\lambda^2} \cdot \frac{\exp[ik(r_2 - r_1)]}{r_1^2 r_2^2} \iiint\int_{-\infty}^{\infty} (x_2 - x_{20}) W_{xx}^0(\rho_{10}, \rho_{20}, 0) \exp\left(ik\frac{\rho_{20}^2 - 2x_2x_{20} - 2y_2y_{20}}{2r_2}\right) \times \exp\left(-ik\frac{\rho_{10}^2 - 2x_1x_{10} - 2y_1y_{10}}{2r_1}\right) dx_{10}dx_{20}dy_{10}dy_{20}, \tag{10}$$

$$W_{zz}(\rho_1, \rho_2, z) = \frac{\exp[ik(r_2 - r_1)]}{\lambda^2 r_1^2 r_2^2} \iint\iint_{-\infty}^{\infty} (x_1 - x_{10})(x_2 - x_{20}) W_{xx}^0(\rho_{10}, \rho_{20}, 0) \times \exp\left(ik\frac{\rho_{20}^2 - 2x_2x_{20} - 2y_2y_{20}}{2r_2}\right) \times \exp\left(-ik\frac{\rho_{10}^2 - 2x_1x_{10} - 2y_1y_{10}}{2r_1}\right) dx_{10}dx_{20}dy_{10}dy_{20}, \tag{11}$$

$$W_{\gamma y}(\rho_1, \rho_2, z) = 0, \quad (\gamma = x, y, z). \tag{12}$$

Substituting Eq. (6) into Eq. (9), the element of CSD can be written as