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Modulational instability in nonlocal media with competing non-Kerr nonlinearities

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ABSTRACT

We investigate analytically and numerically the modulational instability (MI) and propagation properties of light in nonlocal media with competing cubic–quintic nonlinearities where the response functions are assumed to be equal. By using the linear stability analysis, the generic properties of the MI gain spectra are demonstrated for the exponential and rectangular response functions. Special attention is paid to investigate the competition between the spatial scale of the cubic and quintic nonlinearities. For media with exponential response function, we have obtained the range of the wave numbers where instability occurs. It is found that the increase in the absolute value of the quintic nonlinearity suppresses the instability in the regime where the cubic nonlinearity prevails over the quintic one and promotes its development in the opposite case. For media with negative response function, additional MI bands are excited at higher wave numbers when the width of the nonlocal response function exceeds a certain threshold. In the regime where the quintic nonlinearity is dominant, the increase in the absolute value of the quintic coefficient leads to the enhancement of the gain value and the movement of the maximum gain to higher wave numbers. On the other hand, in the case of the predominance of the cubic nonlinearity, the position of the maximum gain bands move to lower wave numbers and MI domain becomes increasingly narrows when the quintic term increases. The numerical simulations fully confirm our analytical results.

1. Introduction

Recently initiated theoretical and experimental studies of nonlocal nonlinearities revealed many novel features in the propagation of spatial solitons including the suppression of their modulational [1] and azimuthal [2] instabilities. Modulational instability (MI) in nonlinear media is a destabilization mechanism that produces a self-induced breakup of an initially continuous wave into localized (solitary wave) structures. This phenomenon was predicted in plasma [3,4], nonlinear optics [5,6], fluids [7], and atomic Bose–Einstein condensates [8,9]. MI of continuous waves can be used to generate ultrahigh-repetition-rate trains of solitonlike pulses [10,11]. It is common knowledge that MI is absent in a defocusing Kerr medium and present as a long-wave instability with a finite bandwidth in a focusing Kerr medium [12].

During the past decade, a novel type of nonlinearity, nonlocal nonlinearity, with the refractive index change of a material at a particular location is determined by the intensity in a certain neighborhood of this location [1], and was shown to stabilize completely the high-dimensional vortex soliton [13]. It appears that nonlocality is an inherent property of thermal media [13,14], nematic liquid crystals [15], atomic vapors [16], and Bose–Einstein condensates [17], etc. The nonlocal nonlinearity also exists in liquid infiltrated photonic crystal fibers [18], which supports the existence of nonlocal gap soliton [19]. Another very general important class of nonlocal materials is materials with quadratic nonlinearity [20], which has been shown that the nonlocal nature of the quadratic nonlinearity can describe soliton pulse compression [21], the exotic X-waves [22], and analytically give the limits of the achievable pulse length [23]. The MI properties of the plane waves of the nonlocal model for $\chi^{(2)}$ materials were investigated in detail in [24]. Nonlocality appears to have a significant effect on propagation of beams and their localization. For instance, it can suppress transverse instability [25] of optical waves and prevent the catastrophic collapse of self-focusing beams in nonlinear

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media [26].

Recently, nonlocal media with competing nonlinearities have drawn much attention [27,28]. Competing nonlinearities occur in systems where few different physical processes contribute to the overall nonlinear response. This is, e.g., the case of Bose–Einstein condensates with simultaneous local and long range bosonic interaction [29] or nematic liquid crystals with comparable thermal and orientational nonlinearities [27]. It has been shown that competing nonlinearities can stabilize many complex soliton structures, which are otherwise unstable in a medium with one type of nonlocal nonlinearity [30,31]. The competing nonlocal nonlinearities can also destabilize dark soliton state [32] and enable coexistence of dark and bright spatial solitons [28]. Nonlinear light properties in nonlocal media with competing nonlinearities were also investigated and it was shown that the interplay of focusing and defocusing nonlocal nonlinearities leads to attraction or repulsion of solitons depending on their separation distance [33]. However, most of the aforementioned studies concentrate on the simplest model of nonlocal Kerr response. The concept of competing nonlinearities has been also discussed in quadratic media with second-order and Kerr nonlinearities [34]. Early works have found that competition between those nonlinearities arrests collapse [35] and stabilizes solitons [36]. By using an exact Floquet technique to find the MI gain spectrum in $\chi^{(2)}$ materials with general quasi-phase matching gratings, it was found that the periodicity can drastically alter the gain spectrum but never completely removes the instability [37]. There are also nonlocal media whose nonlinear responses should take into account potential saturations of the nonlocal nonlinearity, such as atomic vapors [38,39]. Particularly, Mihalache et al. [40] introduced a new phenomenological model for nonlocal media featuring cubic–quintic nonlocal nonlinearities. Since the cubic–quintic nonlinearity can be regarded as a power-law expansion for saturable nonlinearity, it can serve as an approximate model describing beam propagation in nonlocal media with a saturation of the nonlinear response.

In the present work we will study the effect of the quintic nonlinearity on the stability of plane wave. While many previous papers consider only competiting cubic nonlocal media, we will deal here with the case of competing cubic–quintic nonlinearities. We will consider two practically relevant models for the nonlocal systems. We will use the exponential response function as example of response functions with positive-sign bands and the rectangular response function whose Fourier transform has negative-sign bands. The remainder of the article is arranged as follows. In Section 2, we present the model understudy and the stability analysis of the plane wave. In Section 3, the results of MI gain and the nonlinear development of the MI are investigated in non-Kerr media with exponential response function. Section 4 presents the results in non-Kerr media with rectangular response function. Finally, Section 5 concludes the paper.

2. Model and linear stability analysis

The propagation of an optical beam along the z-axis in non-Kerr media with nonlocal nonlinearities is governed by the nonlinear Schrödinger (NLS) equation

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \delta n(I)u = 0, \tag{1}$$

where the nonlinear refractive index change of the media can be represented by the following phenomenological model [40]:

$$\delta n(I) = \alpha_1 \delta n_1(x, I) + \alpha_2 \delta n_2(x, I) = \alpha_1 \int_{-\infty}^{+\infty} R_1(x - x')|u(x', z)|^2\, dx' + \alpha_2 \int_{-\infty}^{+\infty} R_2(x - x')|u(x', z)|^4\, dx'. \tag{2}$$

Here x is the transverse coordinate, $u(x, z)$ is the complex envelope amplitude, α_1 represents the strength of the nonlocal cubic nonlinearity and α_2 is the relative strength of the nonlocal component of the nonlinear response, and its positive (negative) sign represents the self-focusing (self-defocusing) quintic nonlinearity. The form of the convolution integrals represents the nonlocal response functions, which obey the normalization condition $\int_{-\infty}^{+\infty} R_{1,2}(x)\, dx = 1$.

In general, the particular form of the response function is determined by the specifics of the physical process responsible for the nonlinearity of the optical medium. For instance it can be shown that for the reorientational nonlinearity of nematic liquid crystals [41] and general diffusion type nonlinearities [42], the nonlocal response is an exponential. For parametric interaction, the response function can also be periodic in certain regimes of the parameter space [1]. The generic properties of the different types of response functions have been studied by Wyller et al. in terms of MI and it was shown that, in general, all types of localized response functions have the same generic properties, provided their Fourier transform is positive-definite [43]. Following [47], we will use for illustration the exponential response function

$$R_{1,2}(x - x') = \frac{1}{2\sigma_{1,2}} \exp\left(-\frac{|x|}{\sigma_{1,2}}\right), \quad \hat{R}_{1,2}(k) = \frac{1}{1 + \sigma_{1,2}^2 k^2}, \tag{3}$$

as examples of response functions with sign-definite Fourier images, and the rectangular response function

$$R_{1,2}(x) = \begin{cases} \dfrac{1}{2\sigma_{1,2}} & -\sigma_{1,2} \le x \le \sigma_{1,2}, \\ 0 & \text{otherwise}, \end{cases} \quad \hat{R}_{1,2}(k) = \frac{\sin(k\sigma_{1,2})}{k\sigma_{1,2}}, \tag{4}$$

whose Fourier transform has negative-sign bands. Here we have introduced the spatial Fourier transform of the response function as $\hat{R}_{1,2}(k) = \int_{-\infty}^{\infty} R_{1,2}(x)\exp(ikx)\, dx$. The coefficients σ_1 and σ_2 determine the corresponding nonlocality ranges of the cubic and quintic nonlinearities.

The fundamental framework of MI analysis relies on the linear stability analysis, such that a continuous wave (CW) solution is perturbed by a small amplitude or phase perturbation satisfying the condition $|a(z, x)|^2 \ll |P_0|$, and then study whether the perturbation amplitude grows or decays with propagation. The nonlocal NLS equation (1) permits exact CW solutions of the form

$$u(z, x) = \sqrt{P_0}\, \exp(ik_0 x - i\omega_0 z), \tag{5}$$

where P_0, k_0 , and ω_0 are linked through the nonlinear dispersion relation

$$\omega_0 = \tfrac{1}{2}k_0^2 - \alpha_1 P_0 - \alpha_2 P_0^2. \tag{6}$$

Following the procedure listed in [47], we obtained the following equation for the stability of the CW:

$$\lambda^2 = -k^2 P_0\left[\alpha k^2 - \left(\alpha_1 \hat{R}_1(k) + 2\alpha_2 P_0 \hat{R}_2(k)\right)\right], \tag{7}$$

where we have defined the parameter α as

$$\alpha = \frac{1}{4P_0}, \tag{8}$$

k denotes the spatial frequency, $\hat{R}_1(k)$ and $\hat{R}_2(k)$ are the Fourier spectra of $R_1(x)$ and $R_2(x)$. The general dispersion relation