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Entanglement manipulation by atomic position in photonic crystals

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ABSTRACT

We consider two entangled atoms, each of which is embedded in a coherent photonic-band-gap (PBG) reservoir. The effect of the atomic embedded position on the entanglement of the two-atom system is studied. We find that the embedded position of the atom plays an important role in the dynamics of entanglement. The variation of the atomic position can lead to the shift between entanglement sudden death and the entanglement trapping. We also consider the entanglement transfer between different subsystems. Our results could be applied to manipulation of entanglement in nanostructured materials.

1. Introduction

Photonic crystals [1,2] are periodic dielectric structures which can exhibit full PBGs. When atoms are embedded in photonic crystals, it is possible to realize key solid-state quantum information processing (QIP) tasks, such as entangling quantum systems in a controlled fashion [3]. Previous studies have shown that entanglement trapping for two atoms coupled to a PBG reservoir can be realized [4–6], and the entanglement can be controlled by the position of the atomic upper level [7] and the dipole–dipole interaction between atoms [8]. Moreover, the non-Markovian atom–field interaction in PBG reservoirs can lead to controlled entanglement between the atom and the reservoir modes [9].

In this paper we propose a different point of view on controlling the entanglement through changing the relative position of the embedded atom. It originates from the fact that the variation of the atomic position would lead to the change of photon–atom coupling strengths [10], which effects the spontaneous emission spectrum and optical properties of atoms [11]. We consider two entangled atoms, each coupled to a coherent two-band PBG reservoir, which depends on the embedded position of the atom. We highlight the effect of atomic position on the entanglement dynamics of the two atoms. A detailed asymptotic analysis shows that the variation of the atomic position would lead to significant changes of entanglement distribution. When the atomic transition frequency is located at the band edge, the entanglement could change from entanglement sudden death (ESD) to entanglement trapping with little variation of the atomic embedded position. The accompanied dynamics of entanglement among other bipartite subsystems is also studied. Our results would be useful for experimental exploration of controlled entanglement in quantum systems composed of quantum dots or Rydberg atoms in PBG materials.

This paper is organized as follows. The physical model is given in Section 2. In Section 3, the effects of the atomic position on the entanglement dynamics of the two-atom system are studied. In Section 4, we study the entanglement transfer between different subsystems. We summarize our results in Section 5.

2. Physical model

We consider two entangled qubits A and B embedded, respectively, in two uncorrelated double-band photonic crystals a and b. Additionally, we assume that the subsystems Aa and Bb are identical. The qubit can be assumed to be a two-level atom with the ground state $|0\rangle$ and the excited state $|1\rangle$. The Hamiltonian, in the rotating-wave approximation, for each local subsystem is ($\hbar = 1$)

$$H = \omega_0|1\rangle\langle 1| + \sum_l \omega_l b_l^\dagger b_l + \sum_\mu \omega_\mu a_\mu^\dagger a_\mu + i\sum_l (g_l(\mathbf{r})b_l^+|0\rangle\langle 1| - H.c.) + i\sum_\mu (g_\mu(\mathbf{r})a_\mu^+|0\rangle\langle 1| - H.c.), \quad (1)$$

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where ω_0 is the atomic transition frequency, $\mathbf{r}$ is the location of the embedded atom, a_μ ($a_\mu^\dagger$) and b_l ($b_l^\dagger$) are the annihilation (creation) operators for the upper and lower band reservoirs, respectively. The spatial dependence coupling constant can be given by [14,15]

$$g_{\mu(l)}(\mathbf{r}) = \omega_0 d_0 \left(\frac{1}{2\varepsilon_0 \omega_{\mu(l)} V_0}\right)^{1/2} \mathbf{u}_d \cdot \mathbf{E}^*_{\mu(l)}(\mathbf{r}), \tag{2}$$

where V_0 is the quantization volume, $\mathbf{u}_d$ and d_0 are the unit vector and the magnitude of atomic dipole moment. The eigenmodes $\mathbf{E}_{\mu(l)}(\mathbf{r})$ can be characterized by Bloch modes, which varies from point to point within a unit cell of the photonic crystal. Here, we assume that the eigenmodes are [11]

$$\mathbf{E}^*_u(\mathbf{r}) = E_{\mathbf{k}} \cos\theta(\mathbf{r})\mathbf{e}, \tag{3}$$

$$\mathbf{E}^*_l(\mathbf{r}) = E_{\mathbf{k}} \sin\theta(\mathbf{r})\mathbf{e}, \tag{4}$$

where $\theta(\mathbf{r})$ is the angle parameter seen by the atom located at $\mathbf{r}$, $\mathbf{e}$ and $E_{\mathbf{k}}$ are the unit vector and the amplitude of the electric field with wave vector $\mathbf{k}$. From the above equation, we can find that the fields of the double-band reservoir are two coherent modes with phase difference $\pi/2$. Thus, the coupling constants can be re-written as $g_u(\mathbf{r}) \cong g_{\mathbf{k}} \cos\theta(\mathbf{r})$ and $g_l(\mathbf{r}) \cong g_{\mathbf{k}} \sin\theta(\mathbf{r})$ with real constant $g_{\mathbf{k}} = \omega_0 d_0 \left(\frac{1}{2\varepsilon_0 \omega V_0}\right)^{1/2} E_{\mathbf{k}}(\mathbf{u}_d \cdot \mathbf{e})$.

Near the two band edge frequencies, the dispersion relationship is

$$\omega = \begin{cases} \omega_{c_1} + A_1(k-k_0)^2, & \omega \geq \omega_{c_1} \\ \omega_{c_2} - A_2(k-k_0)^2, & \omega \leq \omega_{c_2}, \end{cases} \tag{5}$$

where k_0 is wave number corresponding to the band edge, $\omega_{c_{1(c_2)}}$ is the upper (lower) band edge frequency and $A_j = \omega_{c_j}/k_0^2$ ($j = 1, 2$).

The corresponding band-edge density of states takes the form [16,17]

$$\rho_1(\omega) = \frac{V_0}{(2\pi)^3} \frac{k_0^2}{2\sqrt{A_1}} \frac{\Theta(\omega - \omega_{c_1})}{\sqrt{\omega - \omega_{c_1}}}, \quad \omega \geq \omega_{c_1}, \tag{6}$$

$$\rho_2(\omega) = \frac{V_0}{(2\pi)^3} \frac{k_0^2}{2\sqrt{A_2}} \frac{\Theta(\omega_{c_2} - \omega)}{\sqrt{\omega - \omega_{c_2}}}, \quad \omega \leq \omega_{c_2}, \tag{7}$$

with the Heaviside step function $\Theta(x)$. From the above equations, we can find that the density of states diverges at the edge frequency. Thus, the atom–reservoir interaction within PBG materials is highly non-Markovian [18]. In order to solve the problem of the non-Markovian dynamics, we use the discretization method [19]. The core of this method is to divide the density of modes into two parts: the discrete part which is near the band edge frequencies and the perturbance part which is far from the band edge. More specifically, the density of modes near the band edge is replaced by a finite (but large) number of discrete harmonic oscillators, while the rest of the mode density can be treated perturbatively. For the discrete part, we should obtain the frequencies and the atom-field coupling constants of the discrete oscillators. The differential forms of Eqs. (6) and (7) are

$$\Delta N_1 = \rho_1(\omega)\Delta_1\omega, \tag{8}$$

$$\Delta N_2 = \rho_2(\omega)\Delta_2\omega. \tag{9}$$

For $\Delta N_m = 1$ ($m = 1, 2$), we find $\Delta_1\omega_i = 1/\rho_1(\omega_i)$ and $\Delta_2\omega_j = 1/\rho_2(\omega_j)$ with discrete index i and j. Thus, the frequencies of the discrete modes can be obtained by

$$\omega_{1i+1} = \omega_{1i} + \Delta_1\omega_i, \tag{10}$$

$$\omega_{2j+1} = \omega_{2j} + \Delta_2\omega_j, \tag{11}$$

for upper and lower band reservoir, respectively.

The coupling constant to the discrete modes of the two-band reservoir can be found by integration of Eqs. (8) and (9),

$$\sum g_m^2(\mathbf{r})\Delta N_1 \approx \int_{\omega_{c_1}}^{\omega_{v_1}} |g_u(\mathbf{r})|^2 \rho_1(\omega)\, d\omega, \tag{12}$$

$$\sum g_j^2(\mathbf{r})\Delta N_2 \approx \int_{\omega_{v_2}}^{\omega_{c_2}} |g_l(\mathbf{r})|^2 \rho_2(\omega)\, d\omega, \tag{13}$$

where $\omega_{v_{1(2)}}$ is the upper (lower) limit of the discretized part of the density of states. $|g_u(\mathbf{r})|^2\rho_1(\omega) = (\beta/\pi)[1/\sqrt{\omega - \omega_{c_1}}]$ and $|g_l(\mathbf{r})|^2\rho_2(\omega) = (\beta/\pi)[1/\sqrt{\omega_{c_2} - \omega}]$ [11], where β is the effective coupling between the atom and the PBG reservoir. The detailed evolution of β is shown in Ref. [20]. We thus find

$$g_m(\mathbf{r}) \approx \sqrt{\frac{2\beta}{N\pi}\sqrt{\omega_{v_1} - \omega_{c_1}}} \cos\theta(\mathbf{r}), \tag{14}$$

$$g_j(\mathbf{r}) \approx \sqrt{\frac{2\beta}{N\pi}\sqrt{\omega_{c_2} - \omega_{v_2}}} \sin\theta(\mathbf{r}), \tag{15}$$

where N is the number of discrete modes.

We assume that at time $t=0$, the atom is in the excited state $|1\rangle$ and the two reservoir modes are in the vacuum states $|\tilde{0}_m\rangle$ and $|\tilde{0}_j\rangle$, respectively. The state vector of the system is therefore

$$\begin{aligned} |\varphi(t)\rangle = {} & a(t)e^{-i\omega_0 t}|1, \tilde{0}_m, \tilde{0}_j\rangle \\ & + \sum_m c_m(\mathbf{r}, t)e^{-i\omega_m t}|0, \tilde{1}_m, \tilde{0}_j\rangle \\ & + \sum_j c_j(\mathbf{r}, t)e^{-i\omega_j t}|0, \tilde{0}_m, \tilde{1}_j\rangle, \end{aligned} \tag{16}$$

where the radiation state $|0, \tilde{1}_m, \tilde{0}_j\rangle(|0, \tilde{0}_m, \tilde{1}_j\rangle)$ accounts for the mode of upper (lower) band reservoir with frequency $\omega_{m(j)}$ having one excitation.

The equations for the amplitudes are governed by the Schrödinger equation, and after eliminating [18] the off-resonant modes with frequency $\omega > \omega_{v_1}$ and $\omega < \omega_{v_2}$, we obtain

$$\begin{aligned} \dot{a}(t) = {} & i \sum_{\omega_\mu > \omega_{v_1}} \frac{|g_\mu(\mathbf{r})|^2}{\omega_\mu - \omega_0} a(t) + i \sum_{\omega_l < \omega_{v_2}} \frac{|g_l(\mathbf{r})|^2}{\omega_l - \omega_0} a(t) \\ & - i \sum_{m=1}^{N} g_m(\mathbf{r}) c_m(\mathbf{r}, t) e^{-i(\omega_m - \omega_0)t} - i \sum_{j=1}^{N} g_j(\mathbf{r}) c_j(\mathbf{r}, t) e^{-i(\omega_j - \omega_0)t}, \end{aligned} \tag{17}$$

$$\dot{c}_m(\mathbf{r}, t) = -ig_m(\mathbf{r})a(t)e^{i(\omega_m - \omega_0)t}, \tag{18}$$

$$\dot{c}_j(\mathbf{r}, t) = -ig_j(\mathbf{r})a(t)e^{i(\omega_j - \omega_0)t}. \tag{19}$$

By numerically solving the above set of equations, we shall analyze the population and entanglement dynamics of the two-qubit system.