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Ordinary state-based peridynamics for plastic deformation according to von Mises yield criteria with isotropic hardening

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ABSTRACT

This study presents the ordinary state-based peridynamic constitutive relations for plastic deformation based on von Mises yield criteria with isotropic hardening. The peridynamic force density–stretch relations concerning elastic deformation are augmented with increments of force density and stretch for plastic deformation. The expressions for the yield function and the rule of incremental plastic stretch are derived in terms of the horizon, force density, shear modulus, and hardening parameter of the material. The yield surface is constructed based on the relationship between the effective stress and equivalent plastic stretch. The validity of peridynamic predictions is established by considering benchmark solutions concerning a plate under tension, a plate with a hole and a crack also under tension.

1. Introduction

Structural metals exhibit plastic deformation when loaded beyond their elastic limit. In the absence of cracks, their behavior is well understood from a computational point of view within the classical continuum mechanics. Fracture of such components is often preceded by an extensive plastic deformation. The traditional approaches to predict failure usually employ concepts from linear elastic fracture mechanics (LEFM), which is conceptually limited to materials exhibiting brittle behavior. Therefore, the applicability of fracture toughness as defined by LEFM becomes questionable in the presence of plastic deformation. Furthermore, the assumption of a sharp crack tip in LEFM may no longer be valid due to the presence of plastic deformation. Also, unlike elastic fracture, ductile fracture is inherently a path-dependent process involving irreversible energy dissipation by yielding and fracturing of materials.

Peridynamics (PD), introduced by Silling (2000), is a reformulation of the classical continuum mechanics equations that introduces an internal length scale that is lacking in the classical form of the equations. Peridynamics is based on integro-differential equations as opposed to the partial differential equations of classical continuum mechanics. It is extremely suitable for failure analysis of structures because it allows cracks to grow naturally without resorting to external crack growth laws. An extensive literature survey on peridynamics is given by Madenci and Oterkus (2014). Peridynamics is not limited to linear elastic material behavior. As part of "nonordinary" state-based peridynamics, Taylor (2008) and Foster et al. (2010) considered viscoplastic material behavior. Also, Foster et al. (2011) proposed critical energy density as an alternative critical parameter for such material behavior. However, “nonordinary” state-based models employ constitutive relations that are non-native to PD theory. It is prone to oscillations in the regions of steep gradients such as the crack tip. The source of

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such oscillations is due to the inadequate approximation in the force density vector. Breitenfeld et al. (2014) presented various non-mathematical techniques to reduce these oscillations. Free of such oscillations, Mitchell (2011) presented an ordinary state-based PD model for ideal plastic deformation of materials in the absence of crack propagation.

This study presents an ordinary state-based PD plasticity model in accordance with von Mises yield criteria and isotropic hardening. Also, it presents a peridynamic J-integral based damage criteria to predict crack propagation. Furthermore, it includes an innovative approach to impose nonlocal boundary conditions. The peridynamic predictions concern equivalent plastic stretch and effective stress in a plate with a hole and a crack along with J-integral calculations.

2. Peridynamic equation of motion

The peridynamic equation of motion introduced by Silling (2000) and later generalized by Silling et al. (2007) is a nonlinear integro-differential equation in time and space in the form

$$\rho(\mathbf{x})\,\ddot{\mathbf{u}}(\mathbf{x},t) = \int_H (\mathbf{t}(\mathbf{u}'-\mathbf{u},\mathbf{x}'-\mathbf{x},t) - \mathbf{t}'(\mathbf{u}'-\mathbf{u},\mathbf{x}'-\mathbf{x},t))\,dH + \mathbf{b}(\mathbf{x},t), \tag{1a}$$

which can be discretized as

$$\rho_{(k)}\ddot{\mathbf{u}}_{(k)} = \sum_{j=1}^{N}\left[\mathbf{t}_{(k)(j)}\left(\mathbf{u}_{(j)}-\mathbf{u}_{(k)},\;\; \mathbf{x}_{(j)}-\mathbf{x}_{(k)},\;\; t\right) - \mathbf{t}_{(j)(k)}\left(\mathbf{u}_{(k)}-\mathbf{u}_{(j)},\;\; \mathbf{x}_{(k)}-\mathbf{x}_{(j)},\;\; t\right)\right]V_{(j)} + \mathbf{b}_{(k)} \tag{1b}$$

in which each material point is identified by its coordinates, $\mathbf{x}_{(k)}$, and is associated with an incremental volume, $V_{(k)}$, and a mass density, $\rho\ (\mathbf{x}_{(k)})$. With respect to a Cartesian coordinate system, the material point $\mathbf{x}_{(k)}$ experiences displacement, $\mathbf{u}_{(k)}$, and its location is described by the position vector $\mathbf{y}_{(k)}$ in the deformed state. The displacement and body load vectors at material point $\mathbf{x}_{(k)}$ are represented by $\mathbf{u}_{(k)}(\mathbf{x}_{(k)},\ t)$ and $\mathbf{b}_{(k)}(\mathbf{x}_{(k)},\ t)$, respectively. The family of material point $\mathbf{x}_{(k)}$ is denoted by $H_{\mathbf{x}_{(k)}}$, shown in Fig. 1. Similarly, material point $\mathbf{x}_{(j)}$ interacts with material points in its own family, $H_{\mathbf{x}_{(j)}}$.

As illustrated in Fig. 2, the material point $\mathbf{x}_{(k)}$ interacts with its family of material points, $H_{\mathbf{x}_{(k)}}$, and it is influenced by the collective deformation of all these material points, thus resulting in a force density vector, $\mathbf{t}_{(k)(j)}$,acting at material point$\mathbf{x}_{(k)}$. It can be viewed as the force exerted by material point $\mathbf{x}_{(j)}$. Similarly, material point $\mathbf{x}_{(j)}$ is influenced by deformation of the material points, $H_{\mathbf{x}_{(j)}}$, in its own family.

The integrand in Eq. (1a) does not contain any spatial derivatives of displacements. Thus, it is valid everywhere whether or not displacement discontinuities exist in the material. The region H defining the range of material point $\mathbf{x}$ is specified by δ, referred to as the “horizon”. Also, the material points within a distance δ of $\mathbf{x}$ are called the family of $\mathbf{x}$, $H_{\mathbf{x}}$. The locality of interactions depends on the horizon, and the interactions become more local with a decreasing horizon, δ. Hence, the classical theory of elasticity can be considered a limiting case of the peridynamic theory as the horizon approaches zero

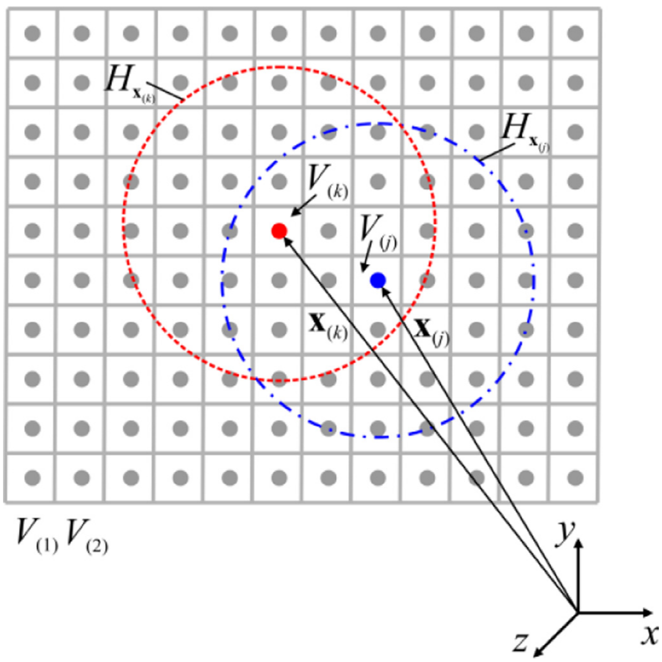

Fig. 1. Peridynamic material points and interaction of points at $\mathbf{x}_{(k)}$ and $\mathbf{x}_{(j)}$.