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Optimal focusing of a beam in a ring vortex

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ABSTRACT

Conventional light focusing, i.e. concentration of an extended optical field within a small area around a point, is a frequently used process in Optics. An important extension to conventional focusing is the generation of the annular focal field of an optical beam. We discuss a simple optical setup that achieves this kind of focusing employing a phase plate as unique optical component. It is assumed that the annular focal field is modulated by an azimuthal phase of integer order q that converts the field in a ring vortex. We first establish the class of beams that being transmitted through the phase plate can be focused into a ring vortex. Then, for each beam in this class we determine the plate transmittance that generates the vortex with the maximum possible intensity, which is referred to as optimal ring vortex.

1. Introduction

Light focusing, one of the processes more employed in optics, is usually realized by a lens. An infinitely small focal point cannot be achieved in free space beam propagation. Even with large aperture lenses, and infinitely extended beams, the minimum focal field size is in the order of half the wavelength [1]. Here we discuss focusing of monochromatic light in an annular focal field, assuming that it is modulated by an azimuthal phase of arbitrary integer order q. The inclusion of the topological charge transforms the focal field in a ring vortex (RV). This type of structured field can be useful in several applications, e.g. optical trapping with orbital angular momentum transference [2–4], lithography [5,6], high-resolution fluorescence microscopy [7], quantum entanglement [8–10], and vortex coronography [11,12].

As occurs in conventional focusing, the generation of an infinitely narrow RV [13–16] is impossible. Therefore, it is important to establish the optimal approximation to this field that can be physically implemented. We consider that the optimal RV generated by a given optical beam is the one with the maximum possible intensity. The maximum intensity in RVs implies other attributes, as narrow transverse section and high intensity gradient that may offer advantages in different applications of such fields.

The generation of an optimal RV at the Fourier domain of a phase diffractive element, which is illuminated by a Gaussian beam (GB), has been recently reported [17]. In the present communication we discuss the simplest method for annular focusing, with arbitrary integer order topological charge, of an input beam. This method employs a phase plate as unique optical component, which modulates the complex amplitude of the beam. The RV is obtained, by free propagation of the modulated beam, at a specific distance from the plate. In Section 2, as first step, we establish the class of beams that can generate a RV using this simple method. Then, we determine the phase plate transmittance required to achieve the optimal annular focusing of the beams in this class. In Section 3 we illustrate the features of optimal RVs, employing both numerical simulations and experiments. In Section 4, we present final remarks and conclusions.

2. Theory

To discuss annular focusing of a beam we refer to the setup depicted in Fig. 1. In this setup, the input beam (B) is passed through a phase plate (PP), and the RV is generated, by free propagation of the transmitted beam, at a distance z from the plate.

For our analysis, the optical fields are expressed in polar coordinates (ξ,ϕ) at the plate plane and (r,θ) at the focal field plane. The complex amplitude of a generic RV, with integer topological charge q, is

$$h(r,\theta) = F(r)\exp(iq\theta), \tag{1}$$

whose radial factor $F(r)$ is specified below. Considering that the RV, with the separable form in Eq. (1), is obtained by free propagation of the field transmitted by the plate, it is easy to prove that this field must have the separable form

$$f(\xi,\phi) = a(\xi)exp[i\beta(\xi)]exp(iq\phi), \tag{2}$$

where the amplitude $a(\xi)$ is a non-negative function and $\beta(\xi)$ is a radial phase function to be determined. This result is obtained

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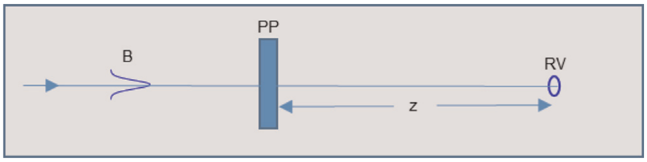

Fig. 1. Simple setup to generate the annular focusing of a beam.

using either the exact or the paraxial scalar field propagation.

Now we assume that the complex amplitude of the beam that illuminates the phase plate is $g(\xi,\phi)=|g(\xi,\phi)|\exp[i\alpha(\xi,\phi)]$, with amplitude $|g(\xi,\phi)|$ and phase $\alpha(\xi,\phi)$. Thus, denoting the phase plate transmittance as $p(\xi,\phi)$, we establish the identity $f(\xi,\phi)=g(\xi,\phi)\ p(\xi,\phi)$. Expressing this relation considering Eq. (2) and the previous formula for $g(\xi,\phi)$ it is easy to show that the complex amplitude of the required input beam is

$$g(\xi,\phi)=a(\xi)\exp[i\alpha(\xi,\phi)], \tag{3}$$

and the transmittance of the phase plate is given by

$$p(\xi,\phi)=\exp[i\beta(\xi)]\exp[-i\alpha(\xi,\phi)]\exp(iq\phi). \tag{4}$$

According to Eq. (3), the family of beams that can be transformed, using the setup of Fig. 1, into the ring vortex with the separable form of Eq. (1), must have an amplitude dependent only in the radial coordinate ξ. However, the phase $\alpha(\xi,\phi)$ in such beams can be an arbitrary function.

The unknown phase $\beta(\xi)$ in Eq. (4) is next specified in order to give desired attributes to the radial factor, $F(r)$, of the RV field. Performing the Fresnel propagation of the field $f(\xi,\phi)$ [Eq. (2)] to a distance z one obtains the field $h(r,\theta)$ [Eq. (1)], whose radial factor is (omitting constants)

$$F(r)=\exp\left(i\frac{kr^2}{2z}\right)\int_0^\infty \xi a(\xi)\exp\left[i\left(\beta(\xi)+\frac{k\xi^2}{2z}\right)\right]J_q\left(2\pi\frac{r}{\lambda z}\xi\right)d\xi, \tag{5}$$

where $k=2\pi/\lambda$ is the wave number and J_q denotes the q-th order Bessel function of the first kind. The integral in Eq. (5) corresponds to the q-th order Hankel transform of the radial function $a(\xi)\exp\{i[\beta(\xi)+k\xi^2/2z]\}$.

Now, let us assume that we desire a RV with radius r_0. We determine the radial phase $\beta(\xi)$ for which this focal field is optimum. From Eq. (5) we can establish the RV intensity at $r=r_0$ as

$$|F(r_0)|^2=\left|\int_0^\infty f_{pos}(\xi)\exp\left[i\left(\beta(\xi)+\frac{k\xi^2}{2z}\right)\right]\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}d\xi\right|^2, \tag{6}$$

where $f_{pos}(\xi)=\xi\ a(\xi)|J_q(2\pi r_0\xi/\lambda z)|$ is a non-negative real function, and 'sgn$\{x\}$' is a binary function, equal to $+1$ for $x\geq 0$, and -1 otherwise. Since the integrand in Eq. (6) is formed by the non-negative function $f_{pos}(\xi)$ multiplied by phase factors, we can obtain the relation [18]

$$|F(r_0)|^2\leq\left(\int_0^\infty f_{pos}(\xi)d\xi\right)^2, \tag{7}$$

where the squared integral represents the upper bound value for $|F(r_0)|^2$. It is straightforward to establish from Eq. (6) that the intensity $|F(r_0)|^2$ achieves the upper bound value if

$$\exp[i\beta(\xi)]=\exp\left(-i\frac{k\xi^2}{2z}\right)\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}. \tag{8}$$

Considering this result in Eq. (4) one obtains the plate phase modulation that generates the optimal RV of radius r_0, which is given by

$$p(\xi,\phi)=\exp\left(-i\frac{k\xi^2}{2z}\right)\exp[-i\alpha(\xi,\phi)]\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}\exp(iq\phi). \tag{9}$$

The phase plate with the transmittance in Eq. (9), illuminated by the input beam $g(\xi,\phi)$ [Eq. (3)], transmits the field $f(\xi,\phi)=a(\xi)\exp(-ik\xi^2/2z)\ \mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}\exp(iq\phi)$. Because of the quadratic phase factor in $f(\xi,\phi)$, the complex amplitude of the RV, at the distance z from the plate, is equivalent to the Fourier transform of the function $a(\xi)\ \mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}\exp(iq\phi)$.

An important input field that belongs to the class defined in Eq. (3) is the GB, whose complex amplitude can be expressed, omitting factors that are independent of ξ, as

$$g(\xi,\phi)=\exp(-\xi^2/w^2)\exp(ik\xi^2/2R), \tag{10}$$

where w is the beam radius and R is the curvature radius of the quadratic phase, at the plate plane. In order to apply the general results to the case of the input GB, it is required to replace the amplitude and the phase in Eq. (3) by $\exp(-\xi^2/w^2)$, and $k\xi^2/2\ R$, respectively. Thus, the plate transmittance that transforms the input GB in an optimal RV, with topological charge q, is

$$p(\xi,\phi)=\exp\left\{-i\frac{k}{2}\left(\frac{1}{R}+\frac{1}{z}\right)\xi^2\right\}\mathrm{sgn}\left\{J_q\left(2\pi\frac{r_0}{\lambda z}\xi\right)\right\}\exp(iq\phi). \tag{11}$$

Note that the quadratic phase factor in Eq. (11), corresponds to the transmittance of a conventional lens, which generates the Fourier transform of the other two factors. On the other hand, the annular form of the focal field, with maximum peak intensity, is allowed by the radial binary phase modulation $\mathrm{sgn}\{J_q(2\pi r_0\xi/\lambda z)\}$. This last factor, together with the azimuthal phase, in both Eq. (9) and Eq. (11), correspond to the phase modulation of the q-th order Bessel beam of radial spatial frequency $\rho_0=r_0/\lambda z$. Two illustrative examples of the phase modulation in Eq. (11), with topological charges $q=0$ and $q=1$, respectively, are depicted in Fig. 2.

Our discussion and results are connected with previous works dealing with the so called perfect vortex [13–16], which is an infinitely narrow RV with arbitrary integer topological charge. It is clear that this ideal field cannot be generated in practice. However, the optimal physically realizable approximation to such ideal RV, employing the optical setup in Fig. 1, is generated by the phase plate whose transmittance is given by Eq. (9), for a generic beam with the complex amplitude specified in Eq. (3), or by Eq. (11), for an input GB. Such phase transmittances can be, in principle, fabricated by lithography on a glass substrate. An attractive option, discussed below, is the use of a phase liquid-crystal (LC) spatial light modulator (SLM).

3. Computational and experimental assessment of optimal RV generators

Next we develop numerical simulations to evaluate optimal

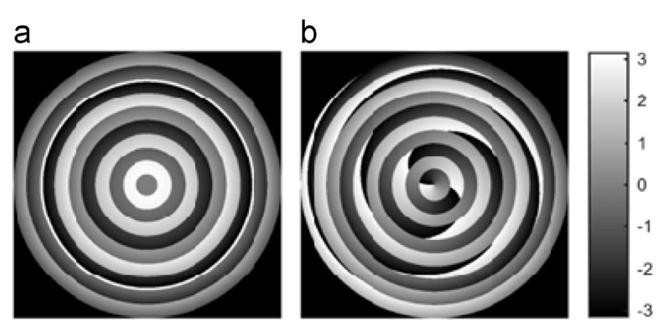

Fig. 2. Central sections in phase modulations of phase plates that generate optimal RVs of topological charges (a) $q=0$, and (b) $q=1$, employing an input Gaussian beam.