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Addition and subtraction operation of optical orbital angular momentum with dielectric metasurfaces

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ABSTRACT

In this work, we propose a simple approach to realize addition and subtraction operation of optical orbital angular momentum (OAM) based on dielectric metasurfaces. The spin–orbit interaction of light in spatially inhomogeneous and anisotropic metasurfaces results in the spin-to-orbital angular momentum conversion. The subtraction system of OAM consists of two cascaded metasurfaces, while the addition system of OAM is constituted by inserting a half waveplate (HWP) between the two metasurfaces. Our experimental results are in good agreement with the theoretical calculation. These results could be useful for OAM-carrying beams applied in optical communication, information processing, etc.

It is well-known that light carries both energy and momentum. The momentum can be divided into two different forms. The first is a spin part associated with polarization state. The second is an orbital part associated with the optical phase profile in a plane orthogonal to the propagation axis [1,2]. Over the past decades, orbital angular momentum (OAM) of light has attracted a great deal of attention due to the potential applications in quantum information processing and quantum cryptography, optical manipulation, quantum computation, and optical communications [3–5]. So far, researchers have studied the properties of vortex beams in detail [6–11] and proposed several alternative approaches to generate OAM-carrying beams by employing optical fiber [12,13], spiral phase plates [14–16], holograms [17–20], inhomogeneous birefringent elements [2,22,21,23], subwavelength gratings [24,25], etc.

Usually, both spin and orbital angular momentum are independent when light propagates in a homogeneous and isotropic transparent medium. Recently, Zhao et al. have shown that spin-to-orbital AM conversion can occur in a homogeneous and isotropic transparent medium when a circularly polarized vortex beam is tightly focused [26]. It is more interesting that inhomogeneous anisotropic media such as *q*-plate constructed with liquid crystal can give rise to spin-to-orbital AM conversion. In the transfer process, the spin angular momentum (SAM) can be entirely converted into its OAM by rationally designing the geometrical distribution of anisotropic medium [2,22,27]. Therefore, inhomogeneous anisotropic media could become an efficient tool to realize the spin-to-orbital conversion.

In this work, a dielectric metasurface is used to act as inhomogeneous anisotropic medium. The dielectric metasurfaces are fabricated by etching space-variant nanoscale waveplatelets inside a fused silica glass with a femtosecond laser. Depending on the amount of deposited energy, the femtosecond laser induces form birefringence in the fused silica glass. One can modify the strength of the birefringence by controlling the writing parameters, especially the polarization of the laser beam [28]. The function of wave-platelet is similar to liquid crystal block of *q*-plate. In comparison to liquid crystal, dielectric metasurface has a higher damage threshold and resolution, which is more convenient to apply in some applications [29–32].

The aim of this paper is to demonstrate a simple and efficient scheme, which can conveniently realize addition and subtraction operation of optical OAM. Firstly, we theoretically investigate the spin–orbit interaction in metasurfaces by using space-variant Jones matrix. We find that two cascaded metasurfaces with π phase retardation can result in the two generated OAMs that are subtracted. When a half waveplate (HWP) is inserted between the two metasurfaces, the output OAMs from the two metasurface add together. In the experimental study, the two adopted metasurfaces both have π phase retardation and $q=1/2$. When a circularly

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polarized beam input the subtraction operation system constituted with the two cascaded metasurfaces, the first metasurface can generate a vortex beam (OAM-carrying beam) with opposite polarization handedness, while the second metasurface can eliminate the vortex (subtract the OAM from the first metasurface). The addition system of OAM is constituted by inserting a HWP between the two metasurfaces. The HWP reverse the polarization handedness of the light emerging from the first metasurface. When the light passes through the second metasurface, the topological charge of vortex changes from $\pm$ 1 to $\pm$ 2, i.e., the vortex number increases to 1. These results could be useful that OAM-carrying beam applies in optical communication, information processing, etc.

1. Theoretical analysis

Metasurfaces are usually structured by etching artificial subwavelength structures in planar dielectric or metallic materials [33,34]. Comparing with metallic metasurfaces, dielectric metasurfaces usually have a higher transmission efficiency of light. In this work, the metasurfaces are fabricated with a fused silica glass. The femtosecond laser is employed to write the nanostructures inside the silica glass. The intense laser irradiation can induce the variation of refractive index of the glass which depends on the amount of deposited energy. It means that we can regard each nanostructure as an artificial uniaxial crystal or waveplate. We assume each wave-platelet of the metasurfaces has a homogeneous phase retardation. It can be easily derived that the Jones matrix of a metasurface is given by

$$\mathbf{M} = \begin{pmatrix} \cos\left(\frac{\Gamma}{2}\right) - i\cos(2\theta)\sin\left(\frac{\Gamma}{2}\right) & -i\sin(2\theta)\sin\left(\frac{\Gamma}{2}\right) \\ -i\sin(2\theta)\sin\left(\frac{\Gamma}{2}\right) & \cos\left(\frac{\Gamma}{2}\right) + i\cos(2\theta)\sin\left(\frac{\Gamma}{2}\right) \end{pmatrix}, \tag{1}$$

where θ is the angle between the fast axis of wave-platelet and the fixed axis x, Γ is the phase retardation of wave-platelet. In this work we adopt the metasurfaces whose phase retardation is equal to π. In particular, the metasurfaces pattern is given by the following expression:

$$\theta = q\phi + \phi_0, \tag{2}$$

where q is an integer or a semi-integer, $\phi = \arctan\left(\frac{y}{x}\right)$, ϕ_0 indicates the initial orientation on the fixed axis x. Considering these specific conditions, Eq. (1) can be then rewritten as

$$\mathbf{M_{MS}} = \begin{pmatrix} \cos(2q\phi) & \sin(2q\phi) \\ \sin(2q\phi) & -\cos(2q\phi) \end{pmatrix}. \tag{3}$$

Here, we have assumed $\phi_0 = 0$.

Let us consider firstly that the metasurface is illuminated by circularly polarized light with SAM $\sigma\hbar$ (where $\sigma = \pm 1$ and + for the left-handed circular polarization and – for the right-handed circular case). Thus, the Jones vector of the input light can be written as follows:

$$\mathbf{E}_{in}(r, \phi) = E_0(r, \phi) \times [1, \sigma i]. \tag{4}$$

At the metasurface output, we obtain field as

$$\mathbf{E_{MS1}} = E_0 e^{\pm i2q\phi}\begin{pmatrix} 1 \\ \mp i \end{pmatrix}. \tag{5}$$

It is seen from Eq. (5) that the output wave is still circularly polarized, but the polarization handedness is contrary to the input wave's. This is all because the retardation of wave-platelet is π. Moreover, the output wave acquires a helical phase with topological charge $\pm$ $2q$ (+ for the left-handed circular polarization input and – for the right-handed circular case). It is notable that the $\pm$ sign of topological charge is determined by the input polarization handedness. In other words, the sign of OAM of the output wave is determined by the input polarization handedness. From the perspective of AM, each input photon carries SAM $\pm\hbar$, while the carried OAM is zero. The total AM is equal to the SAM, i.e., $\pm\hbar$. At the output of metasurface, the output wave has acquired a helical phase $\pm 2q\phi$ (where topological charge of the optical vortex $m = \pm 2q$). Therefore, each output photon carries SAM $\mp\hbar$, and carries OAM $\pm 2q\hbar$. The total AM is then equal to $(\pm 2q \mp 1)\hbar$ and the variation of AM is $2(\pm q \mp 1)\hbar$. It is well-known that AM must be conserved. So the variation of AM must be exchanged with the metasurface. That the spin–orbit interaction occurs in the inhomogeneity is the intrinsic physical cause of the conversion [35].

It needs to be emphasized again that the sign of OAM of the output wave is determined by the input polarization handedness. We have known that the polarization handedness of the output wave has been reversed by the metasurface. Then, if the output wave pass through another metasurface with π phase retardation, the OAM acquired from the second metasurface should have a opposite sign. In order to verify this point, we assume that the second metasurface is illuminated by the circularly polarized vortex beam emerging from the previous metasurface. In a similar mathematical treatment, the Jones vector of resulting beam is

$$\mathbf{E_{MS2}} = E_0 e^{\pm i2(\pm q \mp q')\phi}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}. \tag{6}$$

Here, q' is the topological charge of the second metasurface. Eq. (6) indicates that the light wave emerging from the second metasurface has OAM $2(\pm q \mp q')\hbar$. The OAM acquired from the second metasurface always has an opposite sign with respect to the previous. In other words, the two OAMs which acquire from respectively the two metasurfaces are subtracted. It means that two cascaded metasurfaces can make up a subtraction system for optical OAM. If the second metasurface has the same topological charge as the first one ($q = q'$), the subtraction operation results from zero OAM. Namely, the second metasurface can convert the LCP (RCP) vortex beam to a RCP (LCP) light without helical phase. Especially, the handedness of output beam is also the same as the input. In fact, the function of the first metasurface is to generate a OAM-carrying beam and the second metasurface to realize the subtraction operation of OAM. In the whole process, the AM (including SAM and OAM) do not change when light pass through the two cascaded metasurfaces with π phase retardation and same topological charge. It must be pointed out, however, that a light exchange AM with a single metasurface.

As the above analysis, the sign of OAM is determined by the input polarization handedness. In order to realize the addition operation of optical OAM, we must reverse the polarization handedness of the light wave emerging from the first metasurface. With our optics knowledge, a half waveplate possesses this ability. Now, we assume that a half waveplate is inserted between the two metasurfaces. The output light from the half waveplate is given by

$$\mathbf{E_{HWP}} = E_0 e^{\pm i2q\phi}\mathbf{M_{HWP}}\cdot\begin{pmatrix} 1 \\ \mp i \end{pmatrix} = E_0 e^{\pm i2q\phi}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}, \tag{7}$$

where $\mathbf{M_{HWP}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the Jones matrix for half waveplate. We easily obtain a spin reversal of a photon by exposing the light field to a half waveplate. It is notable that half waveplate only reverse SAM and do not change OAM. Thus the output wave of the second metasurface should be rewritten as follows: