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Theoretical computation of the polarization characteristics of an X-ray Free-Electron Laser with planar undulator

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ABSTRACT

We show that radiation pulses from an X-ray Free-Electron Laser (XFEL) with a planar undulator, which are mainly polarized in the horizontal direction, exhibit a suppression of the vertical polarization component of the power at least by a factor $\lambda_w^2/(4\pi L_g)^2$, where λ_w is the length of the undulator period and L_g is the FEL field gain length. We illustrate this fact by examining the XFEL operation under the steady state assumption. In our calculations we considered only resonance terms: in fact, non-resonance terms are suppressed by a factor $\lambda_w^3/(4\pi L_g)^3$ and can be neglected. While finding a situation for making quantitative comparison between analytical and experimental results may not be straightforward, the qualitative aspects of the suppression of the vertical polarization rate at XFELs should be easy to observe. We remark that our exact results can potentially be useful to developers of new generation FEL codes for cross-checking their results.

1. Introduction

A Free-Electron Laser (FEL) amplifier which starts up from the shot noise in the electron beam is known as a self-amplified spontaneous emission (SASE) FEL. In the SASE case, the amplification process has its origins in the density fluctuations of the electron beam. SASE FELs are capable of producing coherent, tunable FEL radiation down to a fraction of an Angstrom [1,2]. A SASE FEL which operates in the X-ray wavelength range is called XFEL.

With the advent of FELs, X-ray radiation pulses with unprecedented characteristics were made available to the scientific community. Compared to conventional synchrotron radiation sources, X-ray FELs (XFELs) offer an increase in peak brightness of many orders of magnitude as well as ultrashort pulses in the femtosecond time scale. In this paper we consider an additional feature of XFEL pulses, which is useful in many experiments. Namely the fact that the pulses produced by an XFEL with horizontal planar undulator exhibit an extremely small component of the electric field in the vertical direction. In particular, here we will show that for a typical XFEL setup the horizontally polarized component of radiation is greatly dominant, and that only less that one part in a million of the total intensity is polarized in the vertical plane.

The study of XFEL polarization characteristics is obviously deeply related to the problem of electromagnetic wave amplification in XFEL, which refers to a particular class of self-consistent problems. It can be separated into two parts: the solution of the dynamical problem, i.e. finding the motion of the electrons in the beam under the action of given electromagnetic fields, and the solution of the electrodynamic problem, i.e. finding the electromagnetic fields generated by a given contribution of charge and currents. The problem is closed by simultaneous solution of the field equations and of the equations of motion.

Let us consider the electrodynamic problem more in detail. The equation for the electric field follows from Maxwell's equations. One obtains, in Gaussian units:

$$c^2\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial^2\vec{E}}{\partial t^2} - 4\pi\frac{\partial\vec{j}}{\partial t}. \quad (1)$$

With the help of the identity

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} \quad (2)$$

and Poisson equation

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \quad (3)$$

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we obtain the inhomogeneous wave equation for $\vec{E}$

$$c^2\nabla^2\vec{E} - \frac{\partial^2\vec{E}}{\partial t^2} = 4\pi c^2\vec{\nabla}\rho + 4\pi\frac{\partial\vec{j}}{\partial t}. \quad (4)$$

Once the charge and current densities ρ and $\vec{j}$ are specified as a function of time and position, this equation allows one to calculate the electric field $\vec{E}$ at each point of space and time [3]. The current density source provides the main contribution to the radiation field in an FEL amplifier, and the contribution of the charge density source to the amplification process is negligibly small. This fact is commonly known and accepted in the FEL community. However, we have been unable to find a proof of this fact in literature, except book [4] and review [5], which are only the publications, to the authors' knowledge, dealing with this issue.

Due to linearity, without the gradient term the solution of Eq. (4) exhibits the property that the radiation field $\vec{E}$ points in the same direction of the current density $\vec{j}$. An important limitation of such approximation arises when we need to quantify the linear vertical field generated in the case of an XFEL with planar undulator. In the case $\vec{j}$ points in the horizontal direction (for a horizontal planar undulator), according to Eq. (4), which is exact, only the charge term is responsible for a vertically polarized component of the field: if it is neglected, one cannot quantify the linear vertical field anymore.

Similar to the process of harmonic generation, the process of generation of the vertically polarized field component can be considered as a purely electrodynamic one. In fact, the vertically polarized field component is driven by the charge source, but the bunching contribution due to the interaction of the electron beam with the radiation generated by such source can be neglected. This leads to important simplifications. In fact, in order to perform calculations of the radiation including the vertically polarization component one can proceed first by solving the self-consistent problem with the current source only. This can either be done in an approximated way using an analytical model for the FEL process or, more thoroughly, exploiting any existing FEL code. Subsequently, the solution to the self-consistent problem can be used to calculate the first harmonic contents of the electron beam density distribution. These contents enter as known sources in our electrodynamic process, that is Eq. (4). Solution of that equation accounting for these sources gives the desired polarization characteristics.

Analytical descriptions allow for a proper understanding of the physical principles underlying the phenomena under study, and also provide convenient testing of numerical simulation codes. A SASE XFEL is rather difficult to be described in a fully analytical way. In all generality, its radiation can be represented as a non-stationary random process, and its analytical description is further complicated by the fact that the electron bunch combines both the features of the input signal and of the active medium with time-dependent parameters.

Approximations particularly advantageous for our theoretical analysis include the modeling of the electron beam density as uniform, and the introduction of a monochromatic seed signal. Realistic conditions satisfying these assumptions are the use of a sufficiently long electron bunch with a longitudinal stepped profile and the application of a scheme in the SASE mode of operation for narrowing down the radiation bandwidth. In the framework of this model it becomes possible to describe analytically all the polarization properties of the radiation from an XFEL.[1]

The simplicity of our model offers the opportunity for an almost completely analytical description of an XFEL in the linear mode. As remarked above, a complete description of the operation of an XFEL can be performed only with time-dependent numerical simulation codes. Application of the numerical calculations allows one to describe the most general situation, including arbitrary electron beam quality and nonlinear effects. Finding an analytical solution is always fruitful for testing numerical simulation codes. Up to now, in conventional FEL codes, the contribution of the charge source assumed to be negligible small. However, the charge term alone is responsible for the vertically polarized radiation component, which is our subject of interest. Our analytical results for the high-gain linear regime are expected to serve as a primary standard for testing future FEL codes upgrades.

2. XFEL radiation in resonance approximation

As has been discussed in the Introduction, a quantification of the linear vertical field expected in FELs can be made on the basis of an electrodynamical model. We will restrict our attention to FELs driven by planar undulators. A more extended treatment of higher harmonic numbers is conceptually identical to the treatment in this paper, and only differs as regards actual calculations. Proper initial conditions following e.g. from start-to-end simulations are given as input to an FEL self-consistent code, which calculates the electron beam bunching from the interaction of the beam with the first harmonic radiation. As discussed above, in FEL codes the first harmonic is conventionally calculated accounting only for the leading terms under the resonant approximation, which yields perfectly linearly polarized radiation. Therefore, the output of these codes cannot be directly used to quantify the linear vertical field of FELs. However, the electron beam bunching can be inserted into the field equation as sources. A relatively simple electrodynamical model can then be developed in order to calculate corrections to the leading resonant terms, and therefore to calculate the linear vertical field.

Paraxial Maxwell's equations in the space–frequency domain can be used to describe radiation from ultra-relativistic electrons (see e.g. [8]). Let us define temporal Fourier transform pairs as

$$\bar{f}(\omega) = \int_{-\infty}^{\infty} dt f(t) \exp(i\omega t) \leftrightarrow$$
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \bar{f}(\omega) \exp(-i\omega t) \quad (5)$$

and let us call the transverse electric field in the space–frequency domain, i.e. the Fourier transform of the real electric field in the time domain, $\vec{\bar{E}}_\perp(z, \vec{r}_\perp, \omega)$, where $\vec{r}_\perp = x\vec{e}_x + y\vec{e}_y$ identifies a point on a transverse plane at longitudinal position z, $\vec{e}_x$ and $\vec{e}_y$ being unit vectors in the transverse x and y directions. Here the frequency ω is related to the wavelength λ by $\omega = 2\pi c/\lambda$, c being the speed of light in vacuum. From the paraxial approximation follows that the electric field envelope $\vec{\tilde{E}}_\perp = \vec{\bar{E}}_\perp \exp[-i\omega z/c]$ does not vary much along z on the scale of the reduced wavelength $\lambda/(2\pi)$. As a result, the following field equation holds:

[1] Our model can be close to real situations, based on two techniques recently realized at the LCLS. The first is a technique for producing a uniform electron bunch, and is heavily relying on the use of a slotted spoiler foil in the last bunch compressor chicane [6]. The method takes advantages of the high sensitivity of the

(*footnote continued*)
FEL gain process to the transverse emittance of the electron bunch. By spoiling the emittance of most of the long nonuniform bunch while leaving short unspoiled temporal slice, one can produce electron bunch with relatively uniform active medium. The second technique is self-seeding, an active filtering technique allowing to narrow the FEL bandwidth down to almost the Fourier limit [7]. Combination of these two techniques is now widely used at the LCLS.