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The effect of higher order harmonics on second order nonlinear phenomena

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ABSTRACT

A new method which is a combination of the harmonic balance and finite difference techniques (HBFD) is proposed for complete time-harmonic solution of the nonlinear wave equation. All interactions between different harmonics up to an arbitrary order can be incorporated. The effect of higher order harmonics on two important nonlinear optical phenomena, namely, the second harmonic generation (SHG) and frequency mixing is investigated by this method and the results are compared with well-known analytical solutions. The method is quite general and can be used to study wave propagation in all nonlinear media.

1. Introduction

Since the invention of laser, propagation of high intensity optical waves which is substantially affected by the nonlinear properties of the medium has been a topic of high interest. Nonlinearities of the medium lead to optical phenomena such as second harmonic generation, frequency mixing, self-refraction, self-phase modulation and soliton which have all found interesting applications in optoelectronics and optical communications.

In order to study these phenomena, the wave equation must be solved in nonlinear (NL) media. Approximate and simplified closed-form solutions of the NL wave equation for some of these phenomena already exist. These solutions are usually obtained by neglecting higher order harmonics and employing other simplifying assumptions such as slowly varying envelope approximation (SVEA) [1,2] which are only applicable when the nonlinear effects are very weak [3].

Nonlinear Schrödinger equation (NLSE) has been widely used for Soliton propagation and SHG [4–6]. Existence of a slowly varying envelope is the fundamental assumption behind the derivation of NLSE. On the other hand, purely numerical techniques such as finite difference time domain (FDTD) and beam propagation method (BPM) have also been used to study a number of nonlinear problems such as SHG, self-focusing, and Soliton propagation [7–19]. In conventional BPM such as FFT-BPM [15] or FD-BPM [16] the linear and nonlinear parts of the paraxial scalar wave equation are treated separately. In Bidirectional BPM an iterative procedure is employed which is started by solving an independent linear problem to calculate the input/output components of the electric field. Then, energetic exchanges between the two harmonics, which are due to the NL characteristic of the medium, are computed by using these components. Finally, the input/output field components are calculated by considering the energetic exchanges. This iteration stops when the difference between new and old components becomes less than a predefined tolerance [20,21]. Increasing the number of harmonics in this method quickly increases the complexity in calculation of the total energetic exchanges. FDTD can yield more accurate results than those of conventional BPM because it does not use the paraxial approximation. On the other hand, in order to minimize the effects of numerical dispersion while maintaining stability, FDTD requires fine temporal and spatial discretizations which leads to high computational cost of this method [7,8].

In this paper, a new time-harmonic solution for the nonlinear wave equation is presented in which the effects of higher order harmonics up to an arbitrary order are included. The proposed method is inspired by the Harmonic Balance Technique (HBT) which is a well-known method for the analysis of lumped and distributed nonlinear circuits [22]. In this method, all interactions among different harmonics are taken into account while the number of harmonics involved is limited by the user. HBT has also

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been applied to the analysis of nonlinear transmission lines with periodic excitation and formation of shockwaves and solitons in such structures have been reported [23]. The common practice is that the nonlinear transmission line is divided into small segments and each segment is replaced by a lumped element equivalent circuit with nonlinear capacitors. The resulting lumped element circuit is then solved by HBT. The number of nonlinear elements depends on the electrical length of the transmission line, consequently, analyzing a long nonlinear transmission line can become very time consuming by this approach. To overcome this problem, our proposed technique uses finite difference method to solve the nonlinear differential equation in frequency domain. First, the solution is expanded in terms of multiple temporal harmonics with spatially varying coefficients. After balancing the harmonics, a system of nonlinear differential equations for the coefficients is obtained which is solved by the finite difference method. Finally, the Manley–Rowe relations are used to check the balance of power in the medium [24]. The proposed method is called HB-FD technique. It will be used to simulate SHG and frequency mixing in one-dimensional lossless nonlinear media by considering the effects of higher order harmonics. It will be shown that the presence of a higher order harmonic will strongly influence both nonlinear phenomena. It should be stressed that simplifying assumptions such as paraxial approximation and SVEA are not used in the proposed formulation, therefore, not only strong nonlinearity can be considered but the true phase mismatch is also calculated based on the actual dispersion characteristics of the medium. Here the linear and NL parts are not separated and any number of harmonics or combination of waves at different frequencies can be easily incorporated into the solution procedure. These characteristics result in lower complexity, faster solution, and more versatility compared to Bidirectional BPM [20]. Unlike FDTD which can be used to simulate the propagation of narrow pulses of light, the proposed method in its current form can only handle a superposition of finite number of monochromatic waves with different frequencies. Compared to similar methods that employ HBT for modeling wave propagation in nonlinear transmission lines [25], HB-FD technique is faster and can be easily adapted for lossy, dispersive, or inhomogeneous media.

2. Formulation

2.1. Nonlinear wave equation

The wave equation in a NL, homogeneous, and anisotropic medium is derived simply from Maxwell's equations [26]:

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2} \tag{1}$$

in which $c = 1/\sqrt{\epsilon_0\mu_0}$ is the speed of light in vacuum. The polarization vector $\mathbf{P}$ can be separated into linear and NL parts:

$$\mathbf{P} = \epsilon_0\chi^{(1)}\mathbf{E} + \epsilon_0(\overline{\chi}^{(2)}:\mathbf{E}^2 + \overline{\overline{\chi}}^{(3)}\vdots\mathbf{E}^3 + \cdots) = \mathbf{P}_L + \mathbf{P}_{NL} \tag{2}$$

where $\chi^{(1)}$ is the linear first order susceptibility, and $\overline{\chi}^{(2)}, \overline{\overline{\chi}}^{(3)}, \ldots$ are NL higher order susceptibilities [1,4]. When the NL response of the medium is not instantaneous, the successive terms in the above equation should be replaced by convolutions in time domain [26].

2.2. Solution of NL wave equation by HB technique

In this section harmonic balance technique is used to obtain a steady state time-harmonic solution for the NL wave equation. The electric field and polarization are expanded in terms of multiple temporal harmonics with spatially varying coefficients:

$$\mathbf{E} = \sum_{n=1}^{\infty}\mathbf{E}_n(\mathbf{r})e^{j\omega_n t} \tag{3}$$

$$\mathbf{P} = \sum_{n=1}^{\infty}\mathbf{P}_n(\mathbf{r})e^{j\omega_n t} \tag{4}$$

where ω_n stands for various frequencies which include fundamental frequencies, their integer harmonics, and linear combinations of them. $\mathbf{E}_n$ and $\mathbf{P}_n$ are complex vector coefficients of the electric field and polarization at frequency ω_n which, in general, are functions of spatial coordinates. Substituting (3) and (4) into (1) yields

$$\sum_{n=1}^{\infty}\left[\nabla^2\mathbf{E}_n(\mathbf{r}) + \frac{\omega_n^2}{c^2}\mathbf{E}_n(\mathbf{r}) + \mu_0\omega_n^2\mathbf{P}_n(\mathbf{r})\right]e^{j\omega_n t} = 0 \tag{5}$$

After substituting frequency domain descriptions of linear and NL polarizations into (5) and only considering NL susceptibilities of the second and third order (higher order NL susceptibilities are usually insignificant and neglected) we obtain [4]

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[\nabla^2\mathbf{E}_n + \frac{\omega_n^2}{c^2}(1+\chi^{(1)}(\omega_n))\mathbf{E}_n\right]e^{j\omega_n t}\\&\quad+\sum_{n=1}^{\infty}\frac{\omega_n^2}{c^2}\sum_k\sum_l\overline{\chi}^{(2)}(\omega_n;\,\omega_k,\,\omega_l):\,\mathbf{E}_k\mathbf{E}_l e^{j\omega_n t}\\&\quad+\sum_{n=1}^{\infty}\frac{\omega_n^2}{c^2}\sum_p\sum_q\sum_r\overline{\overline{\chi}}^{(3)}(\omega_n;\,\omega_p,\,\omega_q,\,\omega_r)\vdots\mathbf{E}_p\mathbf{E}_q\mathbf{E}_r e^{j\omega_n t} = 0\end{aligned} \tag{6}$$

In order for (6) to hold at all times, coefficients of $\exp(j\omega_n t)$ must be zero. Hence, a NL system of equations is derived which is the time-harmonic equivalent of the NL wave equation:

$$\begin{aligned}&\nabla^2\mathbf{E}_n + \frac{\omega_n^2}{c^2}(1+\chi^{(1)}(\omega_n))\mathbf{E}_n + \frac{\omega_n^2}{c^2}\sum_k\sum_l\overline{\chi}^{(2)}(\omega_n;\,\omega_k,\,\omega_l):\,\mathbf{E}_k\mathbf{E}_l\\&\quad+\frac{\omega_n^2}{c^2}\sum_p\sum_q\sum_r\overline{\overline{\chi}}^{(3)}(\omega_n;\,\omega_p,\,\omega_q,\,\omega_r)\vdots\mathbf{E}_p\mathbf{E}_q\mathbf{E}_r = 0\\&n = 1, 2, 3, \ldots\end{aligned} \tag{7}$$

To demonstrate the basic steps of the proposed method, in this paper, we only consider plane-wave propagation in an unbounded isotropic NL medium. It is assumed that the plane-wave propagates along the z-axis and the electric field has a linear polarization along the x-axis. The medium is homogeneous in transverse plane but its constitutive parameters may vary along the direction of propagation. Therefore, (7) is reduced to a scalar equation:

$$\begin{aligned}&\frac{d^2E_n}{dz^2} + \frac{\omega_n^2}{c^2}(1+\chi^{(1)}(\omega_n))E_n + \frac{\omega_n^2}{c^2}\sum_k\sum_l\chi^{(2)}(\omega_n;\,\omega_k,\,\omega_l)E_kE_l\\&\quad+\frac{\omega_n^2}{c^2}\sum_p\sum_q\sum_r\chi^{(3)}(\omega_n;\,\omega_p,\,\omega_q,\,\omega_r)E_pE_qE_r = 0\\&n = 1, 2, 3, \ldots\end{aligned} \tag{8}$$

in which $E_n(z)$ is the electric field component of the nth harmonic and $\chi^{(i)}$ may be function of z if the medium is inhomogeneous. The system of second order NL differential equations in (8) must be solved numerically.

2.3. Finite difference method

The finite difference method can be used to solve the system of NL differential equations in (7) or (8) in space domain [27]. To solve (8), the z-axis between $z=0$ and $z=L$ (L is arbitrary) is