Optics Communications ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Scattering of electromagnetic plane wave by an impedance strip embedded in homogeneous isotropic chiral medium

Q1 M. Afzaal [a], A.A. Syed [a], Q.A. Naqvi [a,*], K. Hongo [b]

[a] Department of Electronics, Quaid-i-Azam University, Islamabad 45320, Pakistan
[b] 3-34-24, Nakashizu, Sakura City 285-0843, Chiba, Japan

ARTICLE INFO

*Article history:*
Received 25 August 2014
Received in revised form
19 December 2014
Accepted 21 December 2014

ABSTRACT

In this article, Kobayashi potential method has been used to investigate scattering from an impedance strip embedded in a homogenous isotropic chiral medium. The discontinuous properties of the Weber-Schafheitlin's integral have been applied to incorporate the edge conditions. A set of matrix equations are obtained, using the projection properties of Jacobi's polynomials, and solved for unknown expansion coefficients of the fields. Monostatic and bistatic scattering widths have been determined for left and right handed Beltrami field excitations. Effects of variation in values of chirality, incident angle, impedance and width of strip on scattered fields have been investigated. The numerical results of the scattered fields have for various parameters have been produced for comparison.



## 1. Introduction

The scattering of electromagnetic waves is an important area of research because of its myriad and promising applications in atomic physics, geosciences, remote sensing, radar theory, and optics [1–9]. It may be noted that the strip is used as a basic element in the formation of gratings. Scattering from spherical particles and diffraction from gratings are two well-known problems in optics [2–5]. A vast volume of research has surged in this field and numerous techniques have been developed to analyze a variety of scattering problems [10–30]. Some of the well-received techniques are the Physical Optics (PO), Physical Theory of Diffraction (PTD), Wiener-Hopf (WH), Geometrical Theory of Diffraction (GTD), Method of Moment (MoM), T-Matrix, and Maliuzhinet Method etc.

Iwao Kobayashi introduced yet another method to deal with diffraction problems. The attractive feature of the method is that the formulation satisfies both edge and boundary conditions simultaneously by use of the Weber-Schafheitlin's discontinuous integrals. Kobayashi used the proposed method to investigate the problem of an electrified conducting disk [31]. Later on, the method was named as Kobayashi potential (KP) by Sneddon [32]. In the KP method, Eigen function expansion approach is used in the spectral domain similar to MoM but with different formulation [33]. In contrast to MoM method, which is based on integral equations, the KP method is based on dual integral equations [34]. The KP method has been applied in several scattering problems [35–47]. In this investigation, Kobayashi potential method is employed to solve the scattering problem for an impedance strip embedded in isotropic chiral medium. An impedance strip in free space using physical optics theory has been recently treated by [48].

A chiral medium is an optically active medium which has attracted many researchers because of its promising applications in the field of antenna [49–51], radar [52] and waveguide design [53,54]. Recently, propagation of electromagnetic waves through optical waveguide containing chiral nihility metamaterial has been reported [55–57]. A chiral medium can be described by a set of constitutive relations in which electric and magnetic fields are coupled and coupling strength of the fields is described by magnitude of the chirality admittance. Chiral materials may be used to coat objects in order to control the scattering properties of objects as they offer an extra degree of freedom in terms of the chirality parameter [58].

An isotropic chiral medium in Drude–Born–Fedorov representation is characterized by the following constitutive relations [59]:

$$\mathbf{D} = \varepsilon\mathbf{E} + \varepsilon\,\beta\nabla\times\mathbf{E} \tag{1a}$$

$$\mathbf{B} = \mu\mathbf{H} + \mu\,\beta\nabla\times\mathbf{H} \tag{1b}$$

Here **D** and **B** are electric displacement and magnetic induction, respectively. Scalar parameters $\varepsilon$, $\mu$, and $\beta$ are the permittivity,

Q2 * Corresponding author.
*E-mail addresses:* chafzaal007@yahoo.com (M. Afzaal), aqeel@qau.edu.pk (A.A. Syed), qaisar@qau.edu.pk (Q.A. Naqvi), khongo@catv296.ne.jp (K. Hongo).

http://dx.doi.org/10.1016/j.optcom.2014.12.056
0030-4018/© 2014 Published by Elsevier B.V.

permeability and chirality of the isotropic chiral medium, respectively. In this paper, time dependence of electromagnetic fields is taken as $\exp(-j\omega t)$. Electromagnetic fields in chiral medium may be written in terms of Beltrami fields as

$$\mathbf{E} = \mathbf{Q}_1 - j\eta\mathbf{Q}_2 \tag{2a}$$

$$\mathbf{H} = -\frac{j}{\eta}\mathbf{Q}_1 + \mathbf{Q}_2 \tag{2b}$$

In Eq. (2), $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are left and right handed Beltrami fields, respectively, and $\eta = \sqrt{\mu/\varepsilon}$ is the impedance of the isotropic chiral medium. Beltrami fields are known as fields which satisfy the following relations:

$$\nabla \times \mathbf{Q}_{\substack{1\\2}} = \pm\gamma_{\substack{1\\2}}\mathbf{Q}_{\substack{1\\2}} \tag{3a}$$

$$\nabla.\,\mathbf{Q}_{\substack{1\\2}} = 0 \tag{3b}$$

where upper and lower subscripts are for left and right handed Beltrami fields, respectively. Wave number $\gamma_1 = k/(1-k\beta)$ is associated with the left handed Beltrami field whereas the wave number $\gamma_2 = k/(1+k\beta)$ is associated with the right handed Beltrami field and $k = \omega\sqrt{\varepsilon\mu}$.

## 2. Formulation of the problem and KP method

An impedance strip of width $2a$ having negligible thickness is placed in an isotropic chiral medium. It is assumed that the strip is infinite along $z$-axis. Upper and lower surfaces of the strip have impedances $Z_+$ and $Z_-$, respectively. The incident electric field is written below

$$\mathbf{E}^{inc} = \left(-j\sin\phi_o\hat{x} + j\cos\phi_o\hat{y} + \hat{z}\right)Q_{1z}^{inc} - j\eta\left(j\sin\phi_o\hat{x} - j\cos\phi_o\hat{y} + \hat{z}\right)Q_{2z}^{inc} \tag{4}$$

where $Q_{1z}^{inc}$ and $Q_{2z}^{inc}$ are the longitudinal components of the left and right handed Beltrami fields, respectively, and are given as

$$Q_{\substack{1z\\2}}^{inc} = P_{\substack{1\\2}} e^{-j\gamma_{\substack{1a\\2}}(\cos\phi_0 x_a + \sin\phi_0 y_a)} \tag{5}$$

Here, $x_a = x/a$, $y_a = y/a$ and normalized wave numbers are $\gamma_{1a} = a\gamma_1, \gamma_{2a} = a\gamma_2$. Longitudinal component of the incident left and right handed Beltrami fields have amplitudes $P_1$ and $P_2$, respectively, and both fields are making angle $\phi_0$ with positive $x$-axis of the Cartesian coordinate system. Scattered Beltrami fields from an impedance strip with unknown weighting functions are assumed as

$$Q_{\substack{1z\\2}}^{sca} = \int_0^\infty \left(g_{\substack{1c\\2}}(\xi)\cos(x_a\xi) + g_{\substack{1s\\2}}(\xi)\sin(x_a\xi)\right)e^{\sqrt{\xi^2-\gamma_{1a/2}^2}\,y_a}d\xi, \quad y_a > 0 \tag{6a}$$

$$Q_{\substack{1z\\2}}^{sca} = \int_0^\infty \left(h_{\substack{1c\\2}}(\xi)\cos(x_a\xi) + h_{\substack{1s\\2}}(\xi)\sin(x_a\xi)\right)e^{-\sqrt{\xi^2-\gamma_{1a/2}^2}\,y_a}d\xi, \quad y_a < 0 \tag{6b}$$

where $g_{\substack{1c\\2}}(\xi) - h_{\substack{2s\\2}}(\xi)$ are unknown weighting functions and these weighting functions will be determined by imposing boundary conditions. Related boundary conditions for electric and magnetic fields are as follow:

$$E_z^{tot}|_{y_a=0^+} = -Z_+H_x^{tot}|_{y_a=0^+}, \quad |x_a| \le 1 \tag{7a}$$

$$E_z^{tot}|_{y_a=0^-} = Z_-H_x^{tot}|_{y_a=0^-}, \quad |x_a| \le 1 \tag{7b}$$

$$E_x^{tot}|_{y_a=0^+} = Z_+H_z^{tot}|_{y_a=0^+}, \quad |x_a| \le 1 \tag{7c}$$

$$E_x^{tot}|_{y_a=0^-} = -Z_-H_z^{tot}|_{y_a=0^-}, \quad |x_a| \le 1 \tag{7d}$$

$$E_z^{tot}|_{y_a=0^+} = E_z^{tot}|_{y_a=0^-}, \quad |x_a| \ge 1 \tag{7e}$$

$$E_x^{tot}|_{y_a=0^+} = E_x^{tot}|_{y_a=0^-}, \quad |x_a| \ge 1 \tag{7f}$$

$$H_z^{tot}|_{y_a=0^+} = H_z^{tot}|_{y_a=0^-}, \quad |x_a| \ge 1 \tag{7g}$$

$$H_x^{tot}|_{y_a=0^+} = H_x^{tot}|_{y_a=0^-}, \quad |x_a| \ge 1 \tag{7h}$$

In the above equations, 'tot' stands for total field. Edge conditions for an impedance strip are

$$E_t, H_t \to O(1), \quad |x_a| \to 1 \tag{8}$$

here '$t$' stands for tangential components. Following equations with unknown weighting functions are obtained by applying boundary conditions (7a)–(7d)

$$\begin{aligned}&\left(1 - \frac{Z_+}{\eta}\sin\phi_o\right)\left[P_1e^{-j\gamma_{1a}(\cos\phi_0 x_a)} - j\eta P_2e^{-j\gamma_{2a}(\cos\phi_0 x_a)}\right]\\&+\int_0^\infty\left(1 - \frac{jZ_+}{\eta\gamma_{1a}}\sqrt{\xi^2-\gamma_{1a}^2}\right)\left[g_{1c}(\xi)\cos(x_a\xi) + g_{1s}(\xi)\sin(x_a\xi)\right]d\xi\\&-j\eta\int_0^\infty\left(1 - \frac{jZ_+}{\eta\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\right)\left[g_{2c}(\xi)\cos(x_a\xi) + g_{2s}(\xi)\sin(x_a\xi)\right]d\xi = 0\end{aligned} \tag{9a}$$

$$\begin{aligned}&\left(1 + \frac{Z_-}{\eta}\sin\phi_o\right)\left[P_1e^{-j\gamma_{1a}(\cos\phi_0 x_a)} - j\eta P_2e^{-j\gamma_{2a}(\cos\phi_0 x_a)}\right]\\&+\int_0^\infty\left(1 - \frac{jZ_-}{\eta\gamma_{1a}}\sqrt{\xi^2-\gamma_{1a}^2}\right)[h_{1c}(\xi)\cos(x_a\xi) + h_{1s}(\xi)\sin(x_a\xi)]d\xi\\&-j\eta\int_0^\infty\left[\left(1 - \frac{jZ_-}{\eta\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\right)h_{2c}(\xi)\cos(x_a\xi) + h_{2s}(\xi)\sin(x_a\xi)\right]d\xi = 0\end{aligned} \tag{9b}$$

$$\begin{aligned}&\left(1 - \frac{\eta}{Z_+}\sin\phi_o\right)\left[P_1e^{-j\gamma_{1a}(\cos\phi_0 x_a)} + j\eta P_2e^{-j\gamma_{2a}(\cos\phi_0 x_a)}\right]\\&+\int_0^\infty\left(1 - \frac{j\eta}{Z_+\gamma_{1a}}\sqrt{\xi^2-\gamma_{1a}^2}\right)\left[g_{1c}(\xi)\cos(x_a\xi) + g_{1s}(\xi)\sin(x_a\xi)\right]d\xi\\&+j\eta\int_0^\infty\left(1 - \frac{j\eta}{Z_+\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\right)\left[g_{2c}(\xi)\cos(x_a\xi) + g_{2s}(\xi)\sin(x_a\xi)\right]d\xi = 0\end{aligned} \tag{9c}$$

$$\begin{aligned}&\left(1 + \frac{\eta}{Z_-}\sin\phi_o\right)\left[P_1e^{-j\gamma_{1a}(\cos\phi_0 x_a)} + j\eta P_2e^{-j\gamma_{2a}(\cos\phi_0 x_a)}\right]\\&+\int_0^\infty\left(1 - \frac{j\eta}{Z_-\gamma_{1a}}\sqrt{\xi^2-\gamma_{1a}^2}\right)[h_{1c}(\xi)\cos(x_a\xi) + h_{1s}(\xi)\sin(x_a\xi)]d\xi\\&+j\eta\int_0^\infty\left(1 - \frac{j\eta}{Z_-\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\right)[h_{2c}(\xi)\cos(x_a\xi) + h_{2s}(\xi)\sin(x_a\xi)]d\xi = 0\end{aligned} \tag{9d}$$

By using boundary conditions (7e)–(7h), we obtain the following set of equations:

$$\begin{aligned}&\int_0^\infty\left\{g_{1c}(\xi) - h_{1c}(\xi) - j\eta\left(g_{2c}(\xi) - h_{2c}(\xi)\right)\right\}\cos(x_a\xi)d\xi\\&+\int_0^\infty\left\{g_{1s}(\xi) - h_{1s}(\xi) - j\eta\left(g_{2s}(\xi) - h_{2s}(\xi)\right)\right\}\sin(x_a\xi)d\xi = 0\end{aligned} \tag{10a}$$

$$\begin{aligned}&\int_0^\infty\left\{g_{1c}(\xi) - h_{1c}(\xi) + j\eta\left(g_{2c}(\xi) - h_{2c}(\xi)\right)\right\}\cos(x_a\xi)d\xi\\&+\int_0^\infty\left\{g_{1s}(\xi) - h_{1s}(\xi) + j\eta\left(g_{2s}(\xi) - h_{2s}(\xi)\right)\right\}\sin(x_a\xi)d\xi = 0\end{aligned} \tag{10b}$$

$$\begin{aligned}&\int_0^\infty\left\{\sqrt{\xi^2-\gamma_{1a}^2}\left(g_{1c}(\xi) + h_{1c}(\xi)\right) + \frac{j\eta\gamma_{1a}}{\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\left(g_{2c}(\xi) + h_{2c}(\xi)\right)\right\}\cos(x_a\xi)d\xi\\&+\int_0^\infty\left\{\sqrt{\xi^2-\gamma_{1a}^2}\left(g_{1s}(\xi) + h_{1s}(\xi)\right) + \frac{j\eta\gamma_{1a}}{\gamma_{2a}}\sqrt{\xi^2-\gamma_{2a}^2}\left(g_{2s}(\xi) + h_{2s}(\xi)\right)\right\}\sin(x_a\xi)d\xi = 0\end{aligned} \tag{10c}$$



Please cite this article as: M. Afzaal, et al., Optics Communications (2014), http://dx.doi.org/10.1016/j.optcom.2014.12.056