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Lenses to form a longitudinal distribution matched with special functions

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ABSTRACT

We study radially symmetric diffractive optical elements to generate an array of local foci or intensity zeros in the paraxial region by a certain law. The axial distribution is defined by the spatial spectrum of the optical element's radial function, enabling the elements to be called longitudinal-spectrum lenses. The theoretical explanation of the effect is based on the reduction of the Fresnel–Hankel transform to the one-dimensional Fourier transform. The various lenses are analyzed include those generating longitudinal distribution proportional to the Airy and Hermite–Gaussian functions.

1. Introduction

Generating a particular intensity profile along the optical axis (coaxial lines, on-axis foci arrays, optical bottles, optical 'bubble' arrays, etc.) has been important in a number of metrology applications [1–4]; for imaging of extended or moving objects for nondestructive analysis of materials [5–7], as well as for optical micromanipulation [8–11].

The diffractive optical elements (DOEs) to generate a desired longitudinal distribution are designed using both analytical [12–19] and numerical [20–24] techniques.

In this work, we propose radially symmetric DOEs that generate on-axis local foci arrays, with their distribution defined by the spatial spectrum of the optical element's radial function. This property enables the elements to be called longitudinal-spectrum lenses.

The theoretical substantiation of the effect is based on the reduction the Fresnel–Hankel transform to a one-dimensional Fourier transform. Although such an approach has been employed in a number of papers [20,25,26], it was used either to perform the iterative calculation [20] or to find an axial pattern generated by fractal zone plates [25,26].

In this work, we study a wider class of DOEs that enable the generation of definite arrays of foci and/or zero-intensity points, including those proportional to the Airy and Hermite–Gaussian mode distributions.

2. Diffraction in a paraxial region

In the paraxial approximation, the propagation of a radially symmetric light field is described by the following integral Fresnel–Hankel transform:

$$G(\rho, z) = \frac{ik}{z}\exp(ikz)\exp\left(\frac{ik\rho^2}{2z}\right)\int_0^\infty g(r)\exp\left(\frac{ikr^2}{2z}\right)J_0\left(\frac{kr\rho}{z}\right)r\,dr. \tag{1}$$

On the optical axis ($\rho = 0$), Eq. (1) is simplified

$$G(0, z) = \frac{ik}{z}\exp(ikz)\int_0^\infty g(r)\exp\left(\frac{ikr^2}{2z}\right)r\,dr \tag{2}$$

and can be reduced to a one-dimensional (1D) Fourier transform by change of variables [20]. Assuming that the $g(r)$ function is bounded by radius R, the normalized variables can be conveniently introduced:

$$x = \left(\frac{r}{R}\right)^2, \quad u = \frac{R^2}{2\lambda z}. \tag{3}$$

In this case, Eq. (2) takes the form

$$G(u) = i2\pi u\exp\left(\frac{i\pi R^2}{u\lambda^2}\right)\int_0^1 g(x)\exp(i2\pi ux)dx, \tag{4}$$

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which is proportional to the spatial spectrum of the bounded function

$$S(u) = \int_0^T g(x)\exp(i2\pi ux)\mathrm{d}x = \int_{-\infty}^{\infty} g(x)\mathrm{rect}\left(\frac{x - T/2}{T}\right)\exp(i2\pi ux)\mathrm{d}x, \tag{5}$$

where

$$\mathrm{rect}\left(\frac{x}{T}\right) = \begin{cases} 1, & |x| \le T/2, \\ 0, & \text{else}. \end{cases} \tag{6}$$

Making use of Eq. (4), distributions corresponding to the spatial spectrum of the bounded functions can be generated on the optical axis. The intensity on the optical axis

$$I(u) = 4\pi^2 u^2 \left| \int_0^1 g(x)\exp(i2\pi ux)\mathrm{d}x \right|^2 \tag{7}$$

satisfies the radiation condition, because it tends to zero at $u \to 0$ ($z \to \infty$).

Caution needs to be taken when treating the situation at $u \to \infty$ ($z \to 0$), because in this case the paraxial condition can be violated, thus rendering Eq. (1) invalid.

3. Diffraction by a ring aperture

Below, we analyze a simplest case of a uniform input field on the ring aperture:

$$g(r) = \begin{cases} 0, & 0 \le r < r_1, \\ 1, & r_1 \le r \le r_2, \\ 0, & r_2 < r,\ r_2 \le R. \end{cases} \tag{8}$$

Using Eq. (4), we obtain

$$G(u) = au \int_{x_1}^{x_2} \exp(i2\pi ux)\mathrm{d}x = \frac{au}{i2\pi u}[\exp(i2\pi ux_2) - \exp(i2\pi ux_1)] = 2i\exp\left(\frac{i\pi R^2}{u\lambda^2}\right)\exp(i2\pi ux_c)\sin(\pi u\Delta), \tag{9}$$

where $a = i2\pi\exp\left(i\pi R^2/u\lambda^2\right)$, $x_c = (x_1 + x_2)/2$ is the ring median coordinate, and $\Delta = (x_2 - x_1)$ is the ring's width.

In the initial coordinates and in view of Eqs. (7) and (9), the intensity takes the form

$$I(z) = 4\sin^2\left(\pi\frac{r_2^2 - r_1^2}{2\lambda z}\right). \tag{10}$$

As it follows from Eq. (10), the same-intensity optical-axis maxima are found at distances

$$z_n = \frac{r_2^2 - r_1^2}{(2n+1)\lambda}, \quad n = 0,\ 1,\ \ldots \tag{11}$$

Thus, the relative maxima positions are described by the same relationship, which is only scaled due to the aperture width. The outermost maximum from the input plane ($z = 0$) is found at the distance

$$z_0 = \frac{r_2^2 - r_1^2}{\lambda}. \tag{12}$$

The rest maxima are located closer to the input plane, with the inter-maximum distance decreasing as

$$z_n - z_{n+1} = \frac{2\left(r_2^2 - r_1^2\right)}{(2n+1)(2n+3)\lambda} \underset{n\to\infty}{\Longrightarrow} \frac{\left(r_2^2 - r_1^2\right)}{2n^2\lambda}. \tag{13}$$

It stands to reason that at small distances the above approximation is not valid. The aperture width should also be decreased with caution, because in this case the outermost maximum in Eq. (12) will move toward the input plane. The applicability limits of the paraxial approximation were discussed in Ref. [27].

Fig. 1a depicts the simulation results for a ring aperture with radii $r_1 = 1$ mm and $r_2 = 1.5$ mm illuminated by a plane wave of wavelength 532 nm. From Eq. (11), the rounded-off maxima positions at $z_0 \approx 2350$ mm, $z_1 \approx 783$ mm, and $z_3 \approx 470$ mm agree with those derived analytically through the integration of Eq. (2).

The above-derived formulae are also suitable for a circular aperture. In this case, $r_1 = 0$ and $r_2 = R$. Fig. 1b depicts the simulation results for a circular aperture of radius $R = 1$ mm. The maxima are found at $z_0 \approx 1880$ mm, $z_1 \approx 626$ mm, and $z_3 \approx 376$ mm.

For a two-ring aperture

$$g(r) = \begin{cases} 0, & 0 \le r < r_1, \\ 1, & r_1 \le r \le r_2, \\ 0, & r_2 < r < r_3, \\ 1, & r_3 \le r \le r_4, \\ 0, & r_4 < r,\ r_4 \le R \end{cases} \tag{14}$$

the resulting distribution is more complicated

$$G(u) = 2i\exp\left(\frac{i\pi R^2}{u\lambda^2}\right) \times [\exp(i2\pi ux_{c1})\sin(\pi u\Delta_1) + \exp(i2\pi ux_{c2})\sin(\pi u\Delta_2)], \tag{15}$$

where $x_{c1} = (x_1 + x_2)/2$, $x_{c2} = (x_3 + x_4)/2$, $\Delta_1 = (x_2 - x_1)$, $\Delta_2 = (x_4 - x_3)$.

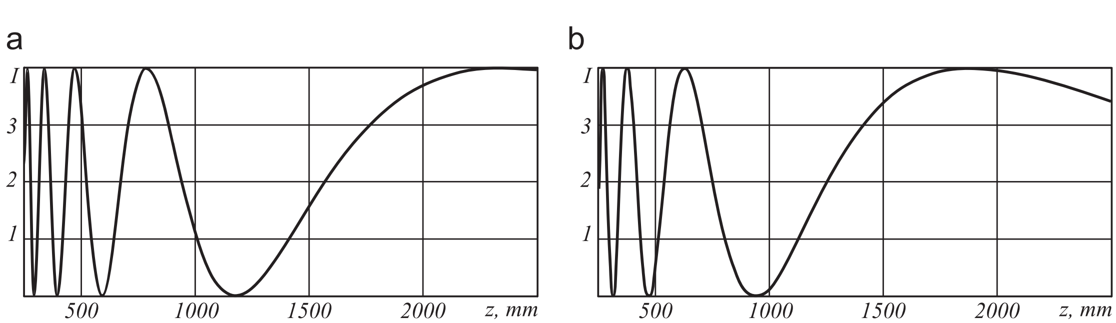

Fig. 1. Intensity along the optical axis for a ring aperture of radii $r_1 = 1$ mm, $r_2 = 1.5$ mm (a) and $r_1 = 0$, $r_2 = 1$ mm (b).