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A micromechanical prediction of localization in a granular material

L. La Ragione [a,*], V.C. Prantil [b], J.T. Jenkins [c]

[a] Dipartimento di Scienze dell'Ingegneria Civile e dell'Architettura, Politecnico di Bari, 70125 Bari, Italy
[b] Department of Mechanical Engineering, Milwaukee School of Engineering, 53202 Milwaukee, WI, USA
[c] School of Civil and Environmental Engineering, Cornell University, 14853 Ithaca, NY, USA

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ABSTRACT

We predict localization in a strained, random aggregate of identical elasto-frictional particles. As triaxial compression proceeds, pairs of particles deform subject to local equilibrium and anisotropy develops because of contact deletion and the dependence of contact stiffness on the average strain. The combination of the deviations of the particle displacements from the average strain and the induced anisotropy results in a relation between increments in average strain and average stress that does not possess the major symmetry. This leads to the possibility of a discontinuity in a component of the incremental strain at a predicted value of the shear strain and a predicted orientation relative to the axis of greatest compression.

1. Introduction

Localization in granular materials has captured the interest of many researchers in past decades because it involves a sharp change between homogeneous and inhomogeneous states. It is a difficult phenomenon to understand and predict because the behavior of the material cannot be represented by the same state variables in the two ranges of deformation. This has been shown in both physical experiments (e.g. Desrues and Viggiani, 2004) and numerical simulations (e.g. Thorton and Zhang, 2006). Consequently, in an analytical treatment, we must distinguish between the pre-peak range, in which the onset of localization may be predicted, and the so-called critical state.

Pioneering works of Rudnicki and Rice (1975), Vardoulakis (1976, 1980), Veermer (1982), and others have predicted localization through a bifurcation analysis based upon an elasto-plastic continuum model, in which the assumption of non-associative plastic flow plays a central role. All such continuum models are based upon assumptions that at the present time can be tested and, eventually, improved, through numerical analysis and sophisticated physical experiments (Andrade et al., 2011).

In particular, discrete element numerical simulations, both dynamic (e.g. Thorton and Zhang, 2006) and static (Wren and Borja, 1997), provide important information for the understanding of localization from the micro-mechanical point of view. Given the shape, size, and the nature of the contacts between particles, such simulations indicate how particles translate, rotate, and rearrange to achieve equilibrium during the deformation, and how the forces transmitted between them evolve. From this information the increments in stress can be related to those of the average strain.

* Corresponding author.
E-mail address: luigi.laragione@poliba.it (L. La Ragione).

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Despite these tools, a theoretical model in which kinematics of contacting particles and the equilibrium and statistics of the aggregate are clearly accounted for is not yet available. Nicot and Darve (2006) provide an important contribution; however, their assumption that the particles deformation follows the average strain limits their final results. We attempt to improve upon the average strain assumption and develop a model that is analogous to the numerical approach of Wren and Borja (1997). In a similar vein, we assume that particle deformations deviate from the uniform strain, we take into account particle equilibrium, and we calculate an elasto-plastic stiffness which has only the two minor symmetries.

Borja and Wren (1995) and Wren and Borja (1997) assume a constant contact stiffness for their two-dimensional disks and, therefore, their anisotropy is associated only with the geometric arrangement of particles. We employ non-linear Hertzian normal contact for an assembly of spheres, and anisotropy is induced by the strain through the contact stiffness. Moreover, we present an analytical solution of the equilibrium equations. This permits an explicit expression for the elasto-plastic stiffness tensor as a function of the applied strain and the average number of contacts per particle. This indicates how the lack of major symmetry in the stiffness tensor can be understood from the micro-mechanical point of view as an interaction between the induced anisotropy and deviations of the particle displacement from the average strain.

We consider a random, aggregate of identical, elasto-frictional spheres with diameter D and number density n that is subjected to an initial isotropic compression followed by a deviatoric shearing without rotation. We predict the onset of a shear band as a bifurcation of a homogeneous deformation, in which a discontinuity of a component of the strain occurs in a predicted direction and value of the shear strain. The bifurcation occurs because the elasto-plastic stiffness tensor, which links increments in stress to increments in total strain, does not possess the major symmetry.

We employ a micro-mechanical analysis to derive the form of this stiffness tensor. In the derivation, we permit the deformation of pairs of particles to depart from the average strain, in order to satisfy local equilibrium, and we incorporate the anisotropy induced in the aggregate by the loading. We do not derive the stress–strain relation along the loading path; but, instead, focus on the incremental response of the stressed aggregate near the maximum shear stress - this is where localization is expected (e.g. Thorton and Zhang, 2006). There, experiments and numerical simulations (e.g. Thornton and Antony, 1998; Radjai et al., 1998) indicate that the component of the incremental contact force normal to the line of centers is negligible, and the incremental response of the aggregate can be idealized as that of a collection of frictionless particles.

We first determine the equilibrium displacements of pairs of spheres that interact through central forces. The analysis focuses on a pair with an arbitrary orientation with respect to the principal axes of strain that interacts with contacting neighbors that are assumed to displace with the average strain. When evaluated using the average strain, the normal component of the contact force varies with the orientation of the pair with respect to the direction of greatest compression and particle contacts in a band orthogonal to this direction are lost. Both the variation of the stiffness and the loss of contacts induce transverse anisotropy into the aggregate. The combination of the deviations of the particle displacements from the

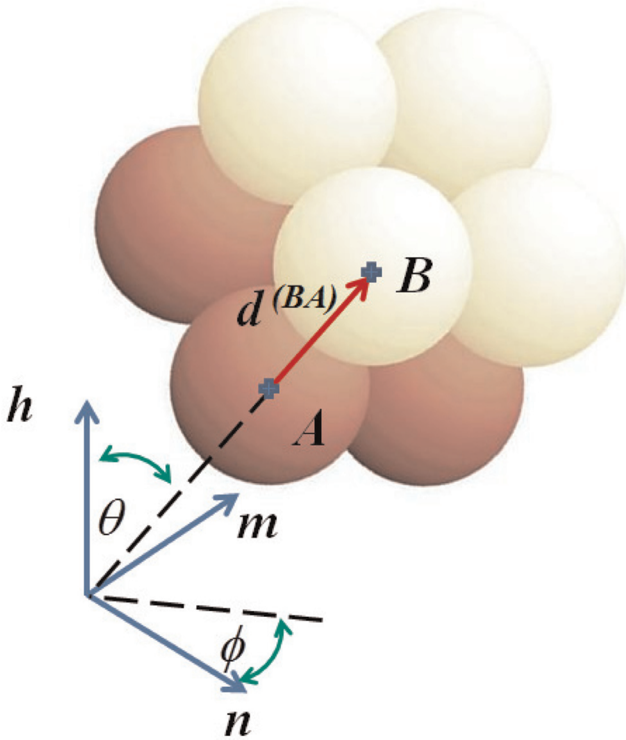

Fig. 1. Typical pair A-B of contacting particles and its orientation with respect to the frame **m**, **n**, **h**.