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Wave vector integral method for design of Fresnel zone plates with different focusing manners

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ABSTRACT

We present an approach for design of Fresnel zone plates (FZPs) with different focusing manners. By re-performing the formula for calculating the half-wave zones of a conventional FZP into wave vector domain, we get a wave vector integral formula, by which the half-wave zones of a generalized FZP could be derived from the wave vector distribution of the expected focusing beams. As examples, we analyzed the situations with, respectively, a circular focusing manner and a line focusing manner, and successfully derived, for the first time to our knowledge, the analytic expressions for calculating the corresponding half-wave zones of the expected FZPs. Some simulations in X-ray domain demonstrated the feasibility of the method and the derived formulas.

1. Introduction

Fresnel zone plates (FZPs) [1] are a typical class of diffractive optical elements, which can perform tasks that are difficult, or even impossible, with conventional refractive optics. For example, focusing and imaging of extreme ultraviolet (EUV) and X-ray radiations at wavelengths from approximately 0.1 to 100 nm are often indispensable in high-resolution X-ray microscopy and spectroscopy [2–8]. However, for these purposes, the use of conventional refractive lenses is not practical, because at these wavelengths all materials are strongly absorbing and the values of the refractive index are very close to 1. FZPs consist of circular diffraction gratings with radially increasing line density, which can diffract and focus the incident X-ray beam into several foci. They offer the advantages of simple fabrication, thin profile and low cost, but the focal spot size of a traditional FZP is approximately the order of the width of the outermost half-zone and so its spatial resolution is limited by the smallest structure the present fabrication technology could reach. Nowadays, the resolution of the best zone-plate optics have reach sub-10 nm at best because of the introduction of nanofabrication technology and use of high-brilliance synchrotron radiation beams, [9,10] which is leading to an increased applications in material science, biology and medicine [11–15].

For further improving the focusing qualities of the FZPs and extending the wavefront transforming ability, recently, some improved structures of FZPs have been developed and applied [16–27]. However, most of the existing FZPs still are designed according to the conventional “point focusing” model, which limit further improvement of the focal spot profile and the focusing trajectory required in different applications.

Here we present a more general formula for design of different types of FZPs with different focusing manner and focal spots. By re-performing the formula for calculating the half-wave zones of a conventional FZP into wave vector domain, we get a wave vector integral formula, by which the half-wave zones of a FZP could be determined from the wave vector distribution of the expected focusing field. As examples, we further analyzed the situations with, respectively, a circle focusing manner and a line focusing manner, and successfully derived, for the first time to our knowledge, two analytical expressions for calculating the corresponding half-wave zones of the expected FZPs.

2. Principle

Let us begin with design of the conventional FZPs based on the ‘point focusing’ manner as shown in Fig. 1(a). The ‘point focusing’ is so called because it assumes that all the designed rays (in other words, the wave vectors) are converged upon a single point, the focal point *F*. Based on such model, the half wave zones of a FZP can be determined by

$$\sqrt{r_m^2 + z_0^2} - \sqrt{r_0^2 + z_0^2} = \frac{\lambda}{2} m \quad (1)$$

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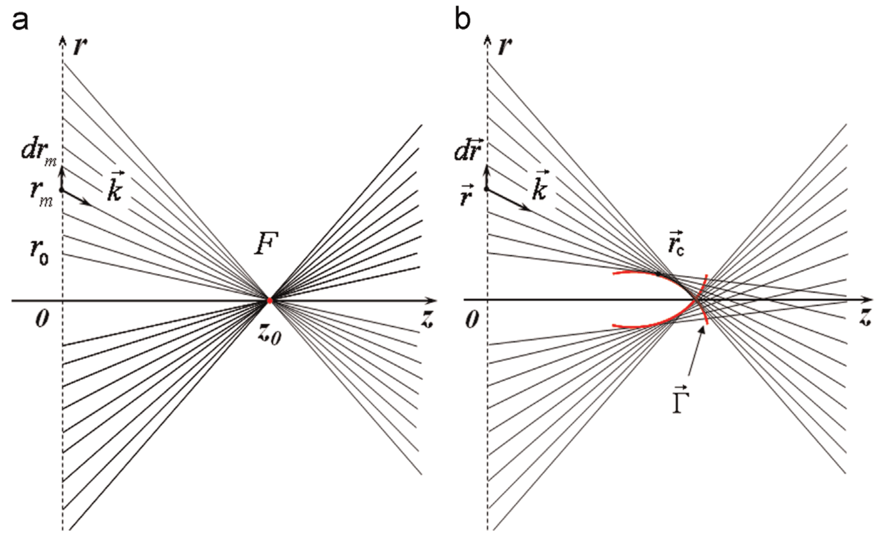

Fig. 1. (a) point focusing manner of a conventional FZP and (b) curve focusing manner of a general FZP.

or

$$k\left(\sqrt{r_m^2+z_0^2}-\sqrt{r_0^2+z_0^2}\right)=m\pi, \tag{2}$$

where $k=2\pi/\lambda$ is the wave number of the beam, λ is the wavelength, r_0 is the start radius, r_m is the radius of the m-th half wave zone, and z_0 is the distance to the focal point, the focal length of the FZP.

Fig. 1(b) shows another more general focusing manner, 'curve focusing', in which the designed wave vectors are focused on or tangent to the curve $\vec{\Gamma}$. Obviously, Eqs. (1) and (2) are not applicable to this case. To find a more general formula suitable for different focusing manners, we try to transform Eq. (2) into wave vector domain. Because the differential of Eq. (2) is equal to

$$\frac{kr_m}{\sqrt{r_m^2+z_0^2}}dr_m=0, \tag{3}$$

Eq. (2) can be rewritten as

$$\int_{r_0}^{r_m}k_r dr=m\pi, \tag{4}$$

in which, $k_r=kr/\sqrt{r^2+z_0^2}$ is exactly equal to the wave vector component of the expected field along the radial coordinate direction on the FZP plane. It can be seen that the half wave zones of the FZPs can be determined by a path integral of the wave vectors. Further, according to the physical meaning of the path integral of wave vectors, we think that Eq. (4) could be extended from the point focusing manners to other focusing situations such as the curve focusing case shown in Fig. 1(b), and it could be further expressed as the more general form in terms of vectors as follows:

$$\int_{\vec{r}_0}^{\vec{r}_m}\vec{k}(\vec{r})\bullet d\vec{r}=m\pi,\ (m=\pm 1,\pm 2,\pm 3,\ldots), \tag{5}$$

here $\vec{k}(\vec{r})$ is the wave vector distribution of the expected beams on the FZP plane, which could be determined by the designed focusing manners. For example, if the focusing wave vectors are supposed to be tangent to a curve $\vec{r}_c(t)=\{x_c(t), y_c(t), z_c(t)\}$, because the tangent vectors of the curve can be generally expressed as

$$\vec{r}_c'(t)=\{x'_c(t), y'_c(t), z'_c(t)\}, \tag{6}$$

where $\vec{r}_c'(t)$is the derivative of the vector $\vec{r}_c(t)$, so the wave vector tangent to the curve can be determined by

$$\vec{k}(\vec{r})=k\vec{r}'_c/\left|\vec{r}'_c\right|. \tag{7}$$

If the designed FZP is circularly symmetric and located in a plane perpendicular to the optical axis (for example, the coordinate z axis in the following situations), the trajectory equation can be expressed as

$$\vec{r}_c(t)=\{r_c(t), z_c(t)\}, \tag{8}$$

and Eq. (5) can be simplified into

$$\int_{r_0}^{r_m}k_r(r)dr=k\int_{r_0}^{r_m}\frac{\partial r_c/\partial t}{\sqrt{(\partial r_c/\partial t)^2+(\partial z_c/\partial t)^2}}dr=\pi m, \tag{9}$$

in which, the coordinate r_c of the curve and the corresponding coordinate r on the FZP plane are related to

$$\frac{r-r_c}{\partial r_c/\partial t}=\frac{z-z_c}{\partial z_c/\partial t}. \tag{10}$$

To verify the feasibility of Eq. (5) in design of FZPs with different focusing trajectory, next we try to determine the half wave zones of a FZP with circular focusing manner (that is, the wave vectors are all tangent to a circular arc) as shown in Fig. 2(a) for the first demonstration.

Suppose the radius of the circular arc is a, and the distance between the circle center located on the z axis and the FZP plane is z_0. Its curve equation can be written as

$$\vec{r}_c=\left\{\sqrt{a^2-(z_c-z_0)^2}, z_c\right\}. \tag{11}$$

Substituting Eq. (11) into Eq. (9), we get

$$k\int_{r_0}^{r_m}\frac{\partial r_c/\partial z_c}{\sqrt{(\partial r_c/\partial z_c)^2+1}}dr=k\int_{r_0}^{r_m}\frac{z_c-z_0}{a}dr=\pi m. \tag{12}$$

Because it can be derived from Eqs. (10) and (11) that (here the FZP is set to be located at the plane of $z=0$)

$$(z_c-z_0)=\frac{ar\sqrt{r^2+z_0^2-a^2}-a^2z_0}{(z_0^2+r^2)}, \tag{13}$$

Eq. (12) can be further expressed as

$$-k\int_{r_0}^{r_m}\frac{r\sqrt{r^2+z_0^2-a^2}-az_0}{(z_0^2+r^2)}dr=\pi m. \tag{14}$$