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Principal component analysis based simultaneous dual-wavelength phase-shifting interferometry

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A B S T R A C T

Combing a sequence of simultaneous phase-shifting dual-wavelength interferograms (SPSDWIs) and principal component analysis (PCA) algorithm, we propose a novel phase retrieval approach in dual-wavelength interferometry. Firstly, for each wavelength, two mutually orthogonal principal component maps are constructed from a sequence of SPSDWIs through using the PCA algorithm, in which SPSDWIs are captured using a monochrome CCD and random and unknown phase shifts. Subsequently, the wrapped phases of each wavelength are obtained directly from the two orthogonal maps by performing the arctangent operation. Finally, an unambiguous phase of an extended synthetic beat wavelength is determined by a simple subtraction between these two wrapped phases. Both, the simulation and the experimental results demonstrate that the proposed approach reveals the simple and convenient performance, faster computing speed and good accuracy.

1. Introduction

Phase-shifting interferometry (PSI) [1], which is a high accuracy and fast speed measuring technique, has been widely utilized in optical surface inspection, three-dimensional shape measurement and biological cell imaging [2–3], among other approaches. In single-wavelength interferometry (SWI), the phase ambiguity problem will appear when the optical thickness of the measured object is larger than the wavelength of the illumination laser [4]. To address this limitation, methods as dual-wavelength interferometry (DWI) [5–7] or multi-wavelength interferometry (MWI) [8,9] have been previously proposed. In DWI, a new unambiguous modulating phase of an extended beat wavelength that in SWI can be synthetically generated as the difference between the two single-wavelength wrapped phases. Therefore, the vertical measurement range can be extended to the micro-range or even millimeters-range. Obviously, the process of obtaining the two single-wavelength wrapped phases from the DWI interferograms plays a crucial step in DWI.

In recent years, there have been reported many single-wavelength wrapped phase retrieval methods in DWI [10–19], In [10–15], spatial Fourier transform (SFT) based approaches are introduced. These methods only require one frame dual-wavelength off-axis interferogram [10–12], or two frames single-wavelength off-axis digital holograms produced by two different wavelength lasers [13–15]. Although these SFT based DWI methods are suitable for real-time and dynamic measurements, their measuring accuracy is greatly affected by the filtering window and noise presence. To address these limitations, Abdelasam et.al propose a kind of single-wavelength phase-shifting (SWPS) based DWI approach [16], in which for each wavelength, a sequence of phase-shifting interferograms are required to retrieve the single-wavelength wrapped phases. Therefore, this measuring process is time-consuming and complicated. In [17], a simultaneous phase-shifting multi-wavelength interferometry based on color CCD recording is proposed, in which a sequence of simultaneous phase-shifting three-wavelength (red–green–blue) interferograms are captured by a color CCD. Then, three groups of single-wavelength phase-shifting interferograms are extracted by direct color separation. Though the measuring accuracy of this approach is very high, it cannot work well when the wavelength difference between the illumination lasers is very small. Additionally, temporal Fourier transform (TFT) based DWI is also developed in [18], in which the phase shifts are produced by a reference mirror moving with a uniform velocity, and a sequence of simultaneous phase-shifting dual-wavelength interferograms (SPSDWIs) are captured by a monochrome CCD. Then the wrapped phases of each single-wavelength are retrieved from the complex amplitude located in the spectral peak by performing TFT for each pixel of SPSDWIs. Obviously, this TFT based DWI approach has a greatly simplified optical setup. Moreover, using this kind of approach, the object

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wave can be recorded with high frequency. However, to ensure the spectral peak separation of the beat frequency spectrum, it is required that the moving distance of the reference mirror and the number of the captured interferograms should be large enough in this approach. Recently, a two-step DWI demodulation approach has been reported [19], which only requires 5-frame SPSDWIs with special phase shifts using a monochrome CCD. This approach has as main limitation the requirement about the special phase shifts.

Phase-shifting interferometry based on principal component analysis (PCA) is a well-known approach which does not need prior knowledge about the phase shifts. Additionally, this method is very fast, as it is not iterative, and robust to noise and then, can obtain the modulating phase with high accuracy [20–23]. In this study, we present a simple, effective and fast phase retrieval approach in DWI combing PCA and SPSDWIs. Following, we will introduce the principle of the proposed approach; then we show a simulation and finally some experimental results are presented. In addition, the results achieved by the proposed approach are compared with those achieved by other DWI approaches.

2. Principle

Assuming two laser beams with wavelength λ_1 and λ_2 go through the same inline phase-shifting interferometry system simultaneously, a sequence of SPSDWIs are captured by a monochrome CCD camera. Thus, the intensity of the nth interferogram can be expressed as the incoherent superposition of two single-wavelength interferograms

$$I_n(x,y) = a(x,y) + b_{\lambda 1}(x,y)\cos[\Phi_{\lambda 1}(x,y) + \delta_{\Lambda 1,n}] + b_{\lambda 2}(x,y)\cos[\Phi_{\lambda 2}(x,y) + \delta_{\Lambda 2,n}]. \tag{1}$$

where x, y represent the spatial coordinates in the CCD plane; $n=0, 1, 2, \ldots, N$ denotes the dual-wavelength phase-shifting interferograms; $a(x,y)$ is the background; $b_{\lambda 1}(x,y)$ and $b_{\lambda 2}(x,y)$ represent the modulation amplitude maps at λ_1 and λ_2, respectively; $\Phi_{\lambda 1}(x,y)$ and $\Phi_{\lambda 2}(x,y)$ are the modulating phases at λ_1 and λ_2, respectively, and finally, $\delta_{\Lambda 1,n}$ and $\delta_{\Lambda 2,n}$ respectively represent the phase shifts of the nth phase-shifting interferogram at λ_1 and λ_2. Thus, Eq. (1) can be rewritten as

$$I_n = a + b_{\lambda 1}\cos\Phi_{\lambda 1}\cos\delta_{\Lambda 1,n} - b_{\lambda 1}\sin\Phi_{\lambda 1}\sin\delta_{\Lambda 1,n} + b_{\lambda 2}\cos\Phi_{\lambda 2}\cos\delta_{\Lambda 2,n} - b_{\lambda 2}\sin\Phi_{\lambda 2}\sin\delta_{\Lambda 2,n}, \tag{2}$$

For convenience, in Eq. (2), we omit the coordinates x and y in Eq. (1). From a sequence of approximately randomly distributed phase-shifts SPSDWIs, the background term can be estimated by

$$\mu_I = \frac{1}{N}\sum_{n=1}^{N} I_n \approx a, \tag{3}$$

After eliminating the background term in Eq. (2), we can obtain

$$I_n - a = \alpha_{\lambda 1n} I_{\lambda 1c} + \beta_{\lambda 1n} I_{\lambda 1s} + \alpha_{\lambda 2n} I_{\lambda 2c} + \beta_{\lambda 2n} I_{\lambda 2s}, \tag{4}$$

where

$$\alpha_{\lambda 1n} = \cos[\delta_{\Lambda 1,n}],\ \beta_{\lambda 1n} = -\sin[\delta_{\Lambda 1,n}],\ \alpha_{\lambda 2n} = \cos[\delta_{\Lambda 2,n}],\ \beta_{\lambda 2n} = -\sin[\delta_{\Lambda 2,n}];$$

$$I_{\lambda 1c} = b_{\lambda 1}\cos[\Phi_{\lambda 1}],\ I_{\lambda 1s} = b_{\lambda 1}\sin[\Phi_{\lambda 1}],\ I_{\lambda 2c} = b_{\lambda 2}\cos[\Phi_{\lambda 2}],\ I_{\lambda 2s} = b_{\lambda 2}\sin[\Phi_{\lambda 2}]$$

From Eq. (4), it is shown that each background-eliminated SPSDWIs can be decomposed into two mutually orthogonal single-wavelength maps $I_{\lambda ic}$, $I_{\lambda is}$ ($i=1,2$). When the fringe number in the interferogram is more than one, there will be the following approximation

$$\sum_{x=1}^{N_x}\sum_{y=1}^{N_y} I_{\lambda ic}(x,y) I_{\lambda is}(x,y) \cong 0, \tag{5}$$

In Eq. (5), $N_x \times N_y$ denotes the size of interferogram and $i=1,2$.

PCA is a statistical approach which utilizes a orthogonal transformation to convert a set of observations of possibility correlated variables into a set of values of linearly uncorrelated variables called principal components [20], so the principal components represent linear combinations of the original variables and the best one dimension subspace based on the least-square principle. If a sequence of N-frame SPSDWIs is captured, and each interferogram is reshaped into a column vector with size M ($M=N_x \times N_y$), then, these interferograms can be expressed in a matrix form as

$$\hat{I} = [I_1, I_2, \cdots, I_n, \cdots, I_N]^T, \tag{6}$$

where I_n denotes the nth interferogram with size of M, and $[\cdot]^T$ represents the transposing operation; Matrix size of $\hat{I}$ in Eq. (6) is $N \times M$. Then, we can obtained the covariance matrix of $\hat{I}$ as

$$C = \left(\hat{I} - \mu_I\right)\left(\hat{I} - \mu_I\right)^T, \tag{7}$$

In Eq. (7), $\left(\hat{I} - \mu_I\right)$ denotes the background-eliminated interferogram and C is a real and symmetric matrix with size of $N \times N$. Clearly, it is easy to find out a group of real eigenvalues and its corresponding orthonormal eigenvectors. According to the matrix theory, the covariance matrix C can be diagonalized as

$$D = ACA^T, \tag{8}$$

where D is a diagonal matrix and A denotes the transformation matrix. In this study, this diagonalization is performed by the singular value decomposition(SVD) algorithm. Then, the principal components of SPSDWIs can be achieved by *Hotelling transform* [24] as

$$Y = A\left(\hat{I} - \mu_I\right), \tag{9}$$

Next, we can construct a new variable Y based on the linear combination of the existing ones, that is $Y = [y_1, y_2, y_3, y_4, \cdots, y_N]^T$, in which each component y_n can be regarded as the coordinates of the orthogonal basis. Observe that by definition the principal components are orthogonal between them, and then $\left\langle y_i, y_j \right\rangle = 0$, with $i \neq j$, where $\langle \cdot, \cdot \rangle$ denotes the inner product operator. Observe from Eq. (5) that if the fringe number in the interferogram is more than one, we can assure that $I_{\lambda ic}$ and $I_{\lambda is}$ with $i=1,2$ are orthogonal. However, in general we have that