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Resonant second harmonic generation in plasma by self-focused twisted beam

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ABSTRACT

The resonant second harmonic generation in the presence of a wiggler magnetic field by twisted laser plasma interaction is surveyed. The wiggler magnetic field provides additional momentum required for the phase matching. The Laguerre–Gaussian modes can be used to control the self-focusing and improve the second harmonic generation in laser plasma interaction. The wave equations for the fundamental and the second harmonic fields have been solved in the paraxial approximation. The generation of the second harmonic considering self-focusing is investigated. Also, the dependence of the second harmonic power on the propagation distance for different values of initial fundamental beam intensity, vortex charge number of the doughnut main beam and plasma density has been obtained.

1. Introduction

Nonlinear interaction of the intense laser beam with plasmas has been the subject of many experimental and theoretical studies in the recent years due to their important role in a large number of high power laser applications, such as X-ray lasers [1–3], laser-driven particle accelerators [4–6] and harmonics generation [7–12]. Generation of harmonic radiation is an important issue of laser plasma interaction and attracts great attention of a number of researchers. It is well known that the electromagnetic beams with a nonuniform distribution of irradiance along the wavefront, such as Gaussian, Hermite–Gaussian and Laguerre–Gaussian (LG) beams, present the phenomenon of self-focusing/defocusing. For a given power of the main beam, the average of the square or cube of the electric vector in the wavefront for nonuniform irradiance distribution is much higher than that for uniform irradiance. Since the magnitude of the generated harmonic is higher in the case of nonuniform irradiance than the uniform irradiance case, there was a need to take into account this nonuniformity in the theory of harmonic generation. The mechanisms associated to the pattern of steep density gradients [13–15], ponderomotive force [16] and parametric decay instability [17,18] for second harmonic generation have been studied in the initial investigations. The second and third harmonic generations for the sake of relativistic nonlinear effect have also been extensively investigated [19–23]. Also, the second harmonic generation by Gaussian mode in the non-relativistic regime of laser plasma interaction is surveyed [24,25]. The applications of harmonic combination frequency generation in the ionosphere have been indicated by Gurevich [26], and recently, the third harmonic generation due to a Gaussian laser beam has been studied in a clustered gas [27] and in a tunnel ionizing gas [28]. Since the Laguerre–Gaussian (LG) modes can be used to control focusing forces and improve the electron bunch quality in laser-plasma accelerators [29], in the present paper, generation of second harmonic in the presence of a wiggler magnetic field due to the propagation of a high irradiance LG beam in plasma is proposed and studied. LG beam with helical wavefront and intensity profile consists of a ring of light beam carrying orbital angular momentum[30,31], can be twisted like a corkscrew about the axis of propagation, has zero intensity at its center and hence also popularly named as twisted light. LG beam can be described by phase singularity on axis with strength l, called the optical vortex charge number, and radial index p [32]. The nonlinearity arising through ponderomotive force caused by the wave magnetic field and the wiggler magnetic field leads to a redistribution of carriers, which modifies the background plasma density profile in a direction transverse to fundamental beam axis [10]. When an intense laser beam acts on plasma, ponderomotive force of the focused beam (caused by the wave magnetic field) pushes the electrons out of the region of high intensity, reducing the local electron density, which leads to the further increase of the plasma dielectric function and consequently an even stronger self-focusing of the laser beam occurs [33]. In consequence, on account of the nonuniform radial distribution of irradiance in the beam in the presence of wiggler field, there is a corresponding distribution of

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electron density and in addition to the fundamental frequency, a second harmonic of frequency is generated. In this paper, the Source Dependent Expansion (SDE) method is used [34], which is a general method for solving the paraxial wave equation [35] with nonlinear source terms. Actually, the SDE method is employed to derive an equation which governs the evolution of the laser beam width.

The remaining part of paper is organized as follow: in the second section, an expression for the wave equation in the presence of a transverse current density caused by wiggler magnetic field by considering ponderomotive force nonlinearity is derived; then, we have followed-up solution of this equation. Also, the second harmonic generation due to the ponderomotive force nonlinear effect is discussed theoretically. The third section is devoted to the numerical investigation for the second harmonic generation and in this section a discussion of results is presented. Finally, in the last section, a brief conclusion has been summarized.

2. Theoretical analysis

2.1. Wave equation

In the presence of the wiggler magnetic field, consider a linearly polarized (in the x-direction) propagation of a high irradiance LG electromagnetic beam of frequency ω in the z-direction in plasma. The propagation of a laser beam in plasma medium and wiggler field is characterized by

$$\nabla^2\vec{E}(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2(\vec{E}(\vec{r},t))}{\partial t^2}=\frac{1}{\epsilon_0 c^2}\frac{\partial\vec{J}(\vec{r},t)}{\partial t} \tag{1a}$$

$$\vec{B}_w=B_w e^{ik_w z}\hat{y} \tag{1b}$$

where $\vec{E}=\vec{E}_1\exp(-i\omega t)+\vec{E}_2\exp(-2i\omega t)$. The origin of second harmonic generation in laser plasma interaction is the radiation of second harmonic current density produced by the force of laser magnetic field in Lorentz force. Here, c is the speed of light in vacuum, ϵ_0 is the vacuum permittivity, k_w is the wiggler wave number, B_w is the amplitude of wiggler field, E_1 and E_2 ($E_2 \ll E_1$) are the amplitudes of the linearly polarized fundamental and second harmonic beams, respectively. It has been assumed in writing Eq. (1) that $\vec{\nabla}(\vec{\nabla}\cdot\vec{E})=0$. The main beam and the second-harmonic electromagnetic wave obey the dispersion relation $k_1=(\omega/c)(1-\omega_p^2/\omega^2)^{1/2}$ and $k_2=(2\omega/c)(1-\omega_p^2/4\omega^2)^{1/2}$, respectively, where $\omega_p=(n_0e^2/\epsilon_0 m_e)$ is the plasma frequency, n_0 is the electron density, e is the charge of electron and m_e is the electron rest mass. Since $k_2>2k_1$, for the phase matching, the difference of momentum can be provided to the second harmonic photon by the wiggler wave number i.e. $k_2=2k_1+k_w$. The periodic wiggler field causes to produce a one-dimensional photonic crystal with wave number k_w along the z-direction. Both k_1 and k_2 are in the z-direction, hence k_2 must be in the z-direction also. The second harmonic is polarized in the x-direction [36]. The laser beam exposes an oscillatory velocity to the electrons i.e. $\vec{V}_1=e\vec{E}/m_e i\omega$ and exerts a ponderomotive force on them. Niti Kant et al. [10] derived the nonlinear current density caused by wiggler magnetic field as the source of a second harmonic wave with the electric field $\vec{E}_2$:

$$\vec{J}_2^{NL}=\frac{n_0e^3B_wE_1^2}{4ic\omega^3m_e^3}\left(\frac{3k_1}{4\omega}+\frac{k_1+k_w}{\omega}\right) \tag{2}$$

As it is expected, the second harmonic current density is proportional to the fundamental frequency intensity, this effect in cooperating with self-focusing causes the oscillation of second harmonic intensity and conversion efficiency versus the normalized propagation distance. Numerical results are presented in Section 3 of the paper.

The second harmonic field also produces a linear current density:

$$\vec{J}_2^{(L)}=-\frac{n_0e^2\vec{E}_2}{2m_e i\omega} \tag{3}$$

Substituting for $\vec{E}$ in Eq. (1), then following Sodha et al. [33] for using expression of nonlinear plasma permittivity, considering the plasma current density and separating $\exp(-i\omega t)$ and $\exp(-2i\omega t)$ terms, one obtains the wave equations governing the fundamental and second harmonic fields:

$$\nabla^2\vec{E}_1(\vec{r},t)+\left[\frac{\omega^2-\omega_p^2}{c^2}+\frac{\omega_p^2}{c^2}\Phi(E_1E_1^*)\right]\vec{E}_1=0 \tag{4a}$$

$$\nabla^2\vec{E}_2(\vec{r},t)+\left[\frac{4\omega^2-\omega_p^2}{c^2}+\frac{\omega_p^2}{c^2}\Phi(E_1E_1^*)\right]\vec{E}_2=-\frac{2i\omega}{c^2}\vec{J}_2^{(NL)} \tag{4b}$$

where

$$\Phi(E_1E_1^*)=(1-e^{-\eta|E_1|^2}) \tag{5}$$

where $\eta=e^2/8m_e\omega^2K_BT_{0e}$, K_B and T_{0e} are the Boltzmann constant and the plasma equilibrium temperature, respectively [33].

For a main LG field E_1, Eq. (4a) can be solved through the explained analysis by reference [34]. The solution is obtained by substituting in Eq. (4a) as

$$\vec{E}_1(r,\phi,z)=\vec{A}_1(r,\phi,z)e^{ik_1z} \tag{6}$$

$k_1=(\omega/c)\sqrt{\epsilon'_{10}(\omega)}$ is the wave number and $\epsilon'_{10}=1-\omega_p^2/\omega^2$ is the linear plasma permittivity of the main beam.

From the second order differential equation for $\vec{A}_1$, one can neglect $\partial^2\vec{A}_1/\partial z^2$ assuming the beam to be slowly converging or diverging. For linearly polarized waves, the vector form of E_1 or A_1 can be taken as a scalar form. On the basis of paraxial approximation, the amplitude of the fundamental electric field E_1 is denoted by $A_1(r,\phi,z)$, which satisfies the following equation:

$$\nabla_\perp^2A_1+2ik_1\frac{\partial A_1}{\partial z}+\frac{\omega_p^2\Phi(|A_1|)}{c^2}A_1=0 \tag{7}$$

Applying the SDE method (algebraic details can be found in [37,38]), one can derive the evolution of the beam width parameter for doughnut LG beams (LG beam with p index equal to zero) in the form

$$\frac{d^2f_1}{dZ^2}=\frac{1}{f_1^3}+\frac{k_1w_0^2\omega_p^2}{f_1\omega^2}\left(\frac{-k_1}{(l+1)!\epsilon'_{10}}\right)\times\int_0^\infty(l+1-\xi_1)\xi_1^le^{-\xi_1}(1-e^{-\eta(A_1A_1^*)_{0,l}})\,d\xi_1 \tag{8}$$

where w_0 is the initial main beam width, $\xi_1=2r^2/w_1^2$ and $f_1=w_1/w_0$, $Z=z/z_R$ ($z_R=kw_0^2/2$ is the Rayleigh length). In Eq. (8), $(A_1)_{0,l}$ on the right-hand side of equation in the integral is complex amplitude of the main doughnut beam:

$$(A_1)_{0,l}=\frac{E_{10}w_0}{w_1(z)}\left(\frac{2}{\pi(l)!}\right)^{1/2}e^{il\phi}e^{i(\psi_1)_{0,l}(z)}\times\left(\frac{\sqrt{2}\,r}{w_1(z)}\right)^{|l|}e^{-[1-i\beta_1(z)]\frac{r^2}{w_1(z)^2}} \tag{9}$$