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A study on quantum discord in Gaussian states

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ABSTRACT

We consider analytically the dynamic behaviors of quantum correlation measured by a quantum discord between two mode Gaussian states coupled to a common squeezed thermal reservoir. We derive the conditions to produce and enlarge quantum discord. If the two modes are initially in factorized squeezed states, we reveal that the thermal bath can not only produce but also amplify the two-mode quantum discord provided that initial squeezing parameters can control properly. Whereas two-modes are initially in a two-mode squeezed vacuum state, whether quantum discord is increased or reduced has a strong relationship with the difference between the squeezing parameters of thermal bath and of considered state.

1. Introduction

For a given bipartite quantum state, it is important to know whether it is entangled or separable. While for pure quantum states these concepts are like two facets of the same gemstone, they correspond to different resources in the general case of mixed states. Namely, while entanglement plays a central role in quantum communication [1], its necessity for mixed-state quantum computation is still unclear [2]. Conversely, several recent studies have shown that separable (that is, not entangled) states, traditionally referred to as "classically correlated," might retain some signatures of quantumness with potential operational applications for quantum technology [3–6]. One such signature is the quantum discord [7], which strives at capturing all the quantum correlations in a bipartite state, including – but not restricted to – entanglement. Quantum discord has been shown to be a property held by almost all quantum states [5] and has recently attracted considerable attention [8–11]. In particular, the vanishing of quantum discord between two systems has been shown to be a requirement for the complete positivity of the reduced subsystem dynamics [12].

There is a large amount of literature to study dynamics of quantum discord for discrete quantum systems. However, many recent protocols for quantum communication and computation are based on continuous variable quantum systems [13–18], and the continuous variable optical system has been used to experimentally realize the unconditional quantum teleportation [19]. Hence, it would be especially valuable to investigate quantum discord for continuous variable quantum systems. However, to our knowledge, there is little literature to discuss quantum discord for continuous variable quantum systems, in particular, the quantum discord dynamics for Gaussian state in noise environment. Although the author Gerardo Adesso et al. and Paolo Giorda Datta et al. have given respectively the definition of the Gaussian quantum discord in Refs. [20,21], and have investigated some properties on Gaussian quantum discord, they have not discussed the dynamical evolution for Gaussian quantum discord in noise environment.

On the other hand, the largest obstacle that quantum information processing has to face is that any realistic quantum systems interact inevitably with their surrounding, which introduce quantum noise into the systems. As a result, the quantum systems will lose their energy (dissipation) and/or coherence (dephasing). Thus, it is of fundamental importance to know the influence of the environment on quantum correlation. Some studies in recent years suggest that the quantum correlation measured by quantum discord is more resistant against the environment than quantum entanglement. Moreover, for some special initial states, quantum correlation in a bipartite quantum system will not be affected by the decoherence environment during an initial time interval [27]. Based on the above-mentioned facts, that the quantum discord is a useful resource for quantum information processing and that quantum systems couple inevitably with their environment, naturally there are two interesting questions:

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(i) Whether the environment can enhance the quantum discord of a quantum systems, i.e., to realize the discord amplification? (ii) Whether we can obtain a stable quantum discord induced by the environment? In order to solve these problems, in this paper, we study the quantum discord dynamics of the two modes Gaussian states under different initial states. We show that it is possible to produce and amplify the quantum discord by the environment-induced interaction under some conditions.

This paper is organized as follows: In Section 2, we present our physical model and study its time evolution. In Section 3, we investigate dynamical behaviors of quantum discord for given initial states. The effects of the difference between the squeezing parameters of studied bath and of a given quantum state on the Gaussian quantum discord are investigated in the section. We summarize our conclusion at the end of this paper.

2. Physical model and solution

Let us begin by introducing the physical system, two non-interacting harmonic oscillators immersed in a common noisy environment. We assume that the two harmonic oscillators have the same frequency, $\omega_1 = \omega_2 = \omega$, and the same interaction with the environment. The Hamiltonian of the total system including the two mode and the environment in the rotating-wave approximation is thus given by

$$H = \hbar \sum_{i=1,2} \omega \hat{a}_i^{+} \hat{a}_i + \hbar \sum_{i=1,2} (\hat{B}\hat{a}_i^{+} + \hat{B}^{+}\hat{a}_i) + \hat{H}_E \tag{1}$$

where $\hat{a}$ and $\hat{a}^{+}$ are the harmonic oscillators' creation and annihilation operators respectively, while $\hat{H}_E$ indicates the free Hamiltonian of the external environment, $\hat{B} = \sum_k g_k \hat{b}_k$, and $\hat{B}^{+} = \sum_k g_k^{*} \hat{b}_k^{+}$, $\hat{b}_k$ and $\hat{b}_k^{+}$ are the corresponding creation and annihilation operators of the kth oscillator with the frequency ω_k, and g_k being the coupling between the environment and the system.

We can use master equation to describe state evolution of a dissipative system. In the interaction picture and Markovian approximation, we can obtain the master equation for the reduced density matrix of the system as

$$\frac{\partial \rho}{\partial t} = \frac{\gamma}{2}\left\{ \sum_{i,j=1}^{2} (N+1)(2\hat{a}_i \rho \hat{a}_j^{+} - \hat{a}_i^{+}\hat{a}_j \rho - \rho \hat{a}_i^{+} \hat{a}_j) + N(2\hat{a}_i^{+} \rho \hat{a}_j - \hat{a}_i \hat{a}_j^{+} \rho - \rho \hat{a}_i \hat{a}_j^{+}) + M(2\hat{a}_i^{+} \rho \hat{a}_j^{+} - \hat{a}_i^{+} \hat{a}_j^{+} \rho - \rho \hat{a}_i^{+} \hat{a}_j^{+}) + M^{*}(2\hat{a}_i \rho \hat{a}_j - \hat{a}_i \hat{a}_j \rho - \rho \hat{a}_i \hat{a}_j) \right\} \tag{2}$$

where the $i=j$ terms in the above-mentioned equation describe the individual dissipations of each mode due to the environment, while the other terms denote the couplings between the modes induced by the common bath. γ is the field-decay rate, N represents the mean photon number of the reservoir, while M denotes a parameter related to the phase correlations of the squeezed reservoir. The Heisenberg uncertainty relation imposes the constraint $|M|^2 \le N(N+1)$. When $M=0$, the reservoir is in a thermal state, then N denotes mean photon number of the thermal reservoir. When $M \neq 0$, the reservoir being a squeezed thermal environment, N denotes mean photon number of the squeezed reservoir.

It is known that a two mode Gaussian states can be described by the characteristic function $\chi(\alpha_1, \alpha_2)$ in Wigner representation. $\chi(\alpha_1, \alpha_2)$ is defined as

$$\chi(\alpha_1, \alpha_2, t) = \mathrm{Tr}[\rho(t) \exp(\alpha_1 \hat{a}_1^{+} + \alpha_2 \hat{a}_2^{+} + h.c)] \tag{3}$$

The dynamic equation reflecting the evolution of states is decided by $\partial\chi/\partial t$. Using the standard operator correspondence, we can transform master equation into a partial differential equation for the characteristic function

$$\frac{\partial}{\partial t}\chi(\alpha, t) = -\frac{\gamma}{2}[(2N+1)\hat{B}_1 + M\hat{B}_2 + \hat{A}_1 + \hat{A}_2]\chi(\alpha, t). \tag{4}$$

Then, formal solution corresponding to Eq. (4) can be written as

$$\chi(\alpha_1, \alpha_2, t) = \exp(\hat{L}t)\chi(\alpha_1, \alpha_2, 0) \tag{5}$$

here $\hat{L} = -(\gamma/2)[(2N+1)\hat{B}_1 + M\hat{B}_2 + \hat{A}_1 + \hat{A}_2]$, and

$$\hat{B}_1 = |\alpha_1|^2 + |\alpha_2|^2 + \alpha_1\alpha_2^{*} + \alpha_1^{*}\alpha_2 \tag{6a}$$

$$\hat{B}_2 = \alpha_1^{*2} + \alpha_2^{*2} + 2\alpha_1^{*}\alpha_2^{*} + \alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \tag{6b}$$

$$\hat{A}_1 = \alpha_1^{*}\frac{\partial}{\partial\alpha_1^{*}} + \alpha_2^{*}\frac{\partial}{\partial\alpha_2^{*}} + \alpha_1\frac{\partial}{\partial\alpha_1} + \alpha_2\frac{\partial}{\partial\alpha_2} \tag{6c}$$

$$\hat{A}_2 = \alpha_1^{*}\frac{\partial}{\partial\alpha_2^{*}} + \alpha_2^{*}\frac{\partial}{\partial\alpha_1^{*}} + \alpha_1\frac{\partial}{\partial\alpha_2} + \alpha_2\frac{\partial}{\partial\alpha_1}. \tag{6d}$$

In Eq. (6), we have assumed that M is real. If $\chi(\alpha_1, \alpha_2, 0)$ in initial state is known, we can solve $\chi(\alpha_1, \alpha_2, t)$. In the next section, we will solve the dynamics for concrete initial states making use of the method.

3. Dynamics of quantum discord of two-mode Gaussian states

In this section, we will investigate dynamics of quantum discord for two-mode Gaussian states. We first recall the definition of quantum discord. It is well known that the total correlations between two subsystems A and B described by a bipartite quantum state $\rho_{AB} = \rho_S$ are generally measured by quantum mutual information given by

$$\mathcal{I}(\rho_S) = S(\rho_A) + S(\rho_B) - S(\rho_S) \tag{7}$$

where $S(\rho) = -\,\mathrm{Tr}(\rho\ \log_2\rho)$ is the von Neumann entropy of density matrix ρ, $\rho_A(\rho_B)$ are the reduced density operators for subsystem $A(B)$. Quantum mutual information is classified as quantum correlation $\mathcal{D}$ and classical correlation $\mathcal{C}$ [24]. And the quantum correlation $\mathcal{D}$ is quantified by the so-called quantum discord [7,25,4]. Then the quantum nature of correlation between two quantum systems is different between quantum mutual information $\mathcal{I}$ and classical correlation $\mathcal{C}$ given by

$$\mathcal{D}(\rho_S) = \mathcal{I}(\rho_S) - \mathcal{C}(\rho_S) \tag{8}$$

which means that to obtain the amount of quantum correlation, one has to find its classical part. The classical correlation $\mathcal{C}(\rho_S)$ is defined as the maximum information about one subsystem ρ_i, which depends on the type of measurement performed on the other subsystem. For a local projective measurement Π_k performed on the subsystem B with a given outcome k, we denote

$$P_k = \mathrm{Tr}\left[(I_A \otimes \Pi_k)\rho_S(I_A \otimes \Pi_k)\right]. \tag{9}$$

as the probability, where I_A is the identity operator for the subsystem A. Then the classical correlation reads