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Linking measures for macroscopic quantum states via photon–spin mapping

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ABSTRACT

We review and compare several measures that identify quantum states that are "macroscopically quantum". These measures were initially formulated either for photonic systems or for spin ensembles. Here, we compare them through a simple model which maps photonic states to spin ensembles. On one hand, we reveal problems for some spin measures to handle correctly photonic states that typically are considered to be macroscopically quantum. On the other hand, we find significant similarities between other measures even though they were differently motivated.

Contents

1. Introduction

The first experiments that triggered the development of quantum mechanics were conducted by relatively simple means. After the theoretical framework has been established, the effort to experimentally verify some much more demanding predictions like entanglement and nonlocality increased significantly. Nowadays, we master experimental techniques that even led to commercial products such as secure communication and true random number generators. Furthermore, it is by now possible to enter the quantum regime of "large" systems; large either in terms of mass, energy or number of involved microscopic constituents. Among other contributions, experimenters brought superconducting devices [1–3] and massive mechanical oscillators [4,5] to the quantum regime; one observed interference effects with giant molecules [6], entangled diamonds [7], cells [8], doped crystals [9] and large spin ensembles [10–12]; we also

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witnessed entanglement between optical modes including hundreds of photons [13,14]. Arguable, all these experiments show quantum behavior in large systems. But how could one compare them? In which sense is one more macroscopically quantum than another one? Answers to these and similar questions would allow us to challenge old but still unsolved problems. One of those is the transition between microscopic and macroscopic domains. How and in which sense do large systems become "classical"—after all, they are composed of microscopic particles that are quantum mechanically in nature? Some proposals try to answer suchlike questions within quantum theory. For instance, the decoherence program [15] provides a mechanism for the loss of quantum correlations that typically becomes stronger for larger systems. Other ideas suggest a solution by extending the theory as it is done, for example, in collapse models [16,17]. Clearly, these efforts are important for problems concerning the validity and interpretation of quantum mechanics. As well, they immediately yield a practical aspect, for instance, in view of efforts to realize large-scale quantum computing.

Against this background, it is somehow astonishing that a commonly accepted framework of "macroscopic quantum physics" is still lacking. The famous gedanken experiment of Schrödinger [18] ponders on the existence of large objects in a superposition of two classical, distinct states like a cat being dead *and* alive. Complementary, Leggett [19,20] argued that, in large systems, there is a difference between an accumulated quantum effect originated on a microscopic scale and a "true" quantum effect on a macroscopic scale. While the former is undoubtedly an experimental challenge due to the complexity and the large number of degrees of freedom, only the latter is supposed to provide insight into the aforementioned problems. Based on these and other contributions [21], many physicists came up with measures to quantify how "macroscopically quantum" a state is [22–30]. Such mathematical definitions potentially provide a clear view on macroscopic quantum effects. Furthermore, an established definition is the basis for theoretical conclusions of, for example, the stability of macroscopic quantum states with respect to noise [22,31] and measurement imperfections [32].

Due to the many proposals on the characterization of macroscopic quantum states, it is clearly necessary to compare those measures in order to understand the similarities and differences. First attempts have been made in Ref. [28], where several measures suitable for spin measures [22,23,25,26,28] have been classified. Another work [33] applies some measures [21,25,26,28] to a specific multi-mode photon state. The ultimate goal is to provide a general framework for macroscopic quantum effects that covers all important physical systems. With this, one is able to directly compare different systems. For instance, one could then compare experiments on trapped ions with massive objects in the superposition of spatial positions (see additional remarks in Section 5).

In this paper, we aim to continue this research line by bridging measures that were formulated for spin systems [22,23,25,26,28] and for single photonic modes [27,30] (some of them are valid for both systems). To this end, we use a simple model of a photon–spin mapping, in particular, the absorption of a photonic state into a spin ensemble. Under the assumption that the properties of the photonic state are completely mapped to the spin ensemble, we have a tool at hand to analyze and compare in which sense states are macroscopically quantum according to different measures.

In the following, we draw some conclusions for several measures based on this mapping. As we will see later, it is necessary that the mean photon number, N, of the considered state is much smaller than the number of the spins in the ensemble, M. After the mapping, N corresponds to the excitation level of the spins. In this regime, we observe that some measures for spin states behave differently than in the case where N is comparable with M, which is the regime where they have been studied so far. Apparently, through this work we also understand better the present proposals and learn about the implications of their initial intuition.

On the other side, we find that there are tight mathematical connections between certain measures, even though the physical motivation for introducing them is apparently very different. We conclude therefore that, at least partially, there exists already some consensus on the characterization of macroscopic quantum states among the present proposals.

This paper is structured as follows. In Section 2, we set the basic nomenclature and summarize the existing proposals in the field of macroscopic quantum states. We also review some established implications on the stability. In Section 3, we introduce and elaborate on the model for the photon–spin interaction we use to link different measures. Some implications are discussed in Section 4. Conclusions and outlook are given in Section 5.

2. Review of measures for macroscopic quantum states

In this section, we first clarify some subtle but important points for the discussion of macroscopic quantum physics (Section 2.1). Then, in Sections 2.2 and 2.3, we give a rough overview on some measures for macroscopic quantum states that have been proposed so far. In Section 2.4, we discuss some implications on the stability in the presence of noise and imperfections.

2.1. Preliminary discussion

Common goal of the measures: The common feature of all works that are considered in this paper is to identify among all quantum states those that are *macroscopically quantum*. This is done by defining a function $f(\psi) \geq 0$ (some proposals are even defined for mixed states). The larger the $f(\psi)$, the more the macroscopically quantum $|\psi\rangle$ is. Often, f is called the *effective size* of $|\psi\rangle$. The *qualitative* distinction between macroscopic and non-macroscopic quantum states based on f is to some extent arbitrary.

System size: All published proposals agree that a quantum state can only be macroscopically quantum if the respective system size is in some sense "large". For systems composed of microscopic particles, it is necessary to have many constituents. If one considers one (or few) bosonic modes, we require to have high excitation numbers or high masses. The exact values for having a "large" system are not crucial for the present discussion. As we are concerned with spin and photonic systems in this paper, the *system size* is defined as the number of spin-$\frac{1}{2}$ particles, M, or the mean photon number, N, respectively.

Schrödinger-cat state vs. macroscopic quantum state: A first distinction of the current literature can be made by the basic form of the states considered to be macroscopic. Some authors [23,25,26,30] consider superpositions of two (or a few) "classical" states like

$$|\psi\rangle \propto |\psi_0\rangle + |\psi_1\rangle. \quad (1)$$

In a simplified way, one can say that one seeks a mathematical definition for the verbal characterization that the states $|\psi_0\rangle$ and $|\psi_1\rangle$ are "macroscopically distinct" [19]. In the remainder of this paper, we call macroscopic superpositions of the form (1) a *Schrödinger-cat state*.

On the other side, some proposals [22,24,27–29] do not require a specific form of the quantum state and may even allow for mixed states. States that are macroscopically quantum due to these definitions are here called *macroscopic quantum states* (see examples below). If any confusion is excluded, we also use this term as an umbrella term that includes the concept of a Schrödinger-cat state.